Risk management - Exam

- The exam lasts two hours.

- Do not forget to write your name on all pages.

- Any document or calculator is forbidden.

- Justify your answers and answer clearly.

Last name:
First name:

Performance and risk of a portfolio

1. Enumerate and explain the statistical properties that we generally observe in financial time series as well as the consequences in risk management. Be as exhaustive as possible.

2. Explain the concept of copula and how copulas are used in risk management. Cite three different copulas and give their respective characteristics, advantages and drawbacks in finance.

3. Propose two different approaches for modeling credit risk and explain how you would choose which one to use in a given context.

A bank suggests the follow model for multivariate stock returns:

$$r_{t+1} \equiv \begin{pmatrix} r_{1,t+1} \\ r_{2,t+1} \\ \dots \\ r_{N,t+1} \end{pmatrix} = \epsilon_{t+1}$$

with $\epsilon_{t+1} \mid \mathcal{F}_t \sim \mathcal{N}(\mu, \Omega)$ i.i.d., $\mu \in \mathbb{R}^N$, $\Omega \in \mathbb{R}^{N \times N}$, and N the number of stocks considered.

4. Explain in details how this model probably fails to describe the data accurately.

5. Explain what is the Value at Risk (VaR) and the Expected Shortfall (CVaR). Describe the consequences of the model above in terms of VaR and CVaR.

6. Propose modifications to improve the fit of the model to data, and justify your assumptions.

We now want to determine the risk of the portfolio of stocks using simulation. We suppose that marginal returns follow a normal distribution $\mathcal{N}(\mu_i, \sigma_i^2)$ for $i = 1, \dots, N$, with μ_i and σ_i constants.

7. The bank hesitates between a parametric copula and a nonparametric copula. Explain the consequences of each alternative.

8. The bank decides in favor of a parametric copula of the form $C(u_1, \cdots, u_N; \theta)$ where θ is a vector of parameters. Give a consistent estimator for each of the parameters μ_i, σ_i and θ.

9. Give a detailed algorithm (in pseudo-code or Matlab) implementing the respective estimators of the previous point.

10. Bonus: Explain how you would use extreme value theory to model the tails of the marginal distributions. Could you still use the copula in this case? Justify.

11. Bonus: Explain the main differences between Matlab and C++ languages, and how you would choose which one to use in a specific context.