# Factors and Risk Premia in Individual International Stock Returns*

Ines Chaieb†, Hugues Langlois‡, and Olivier Scaillet§

July 13, 2020

**ABSTRACT**

We propose an estimation methodology tailored for large unbalanced panels of individual stock returns to study the factor structure and expected returns in international stock markets. We show that the local market is necessary to capture the factor structure in both developed and emerging markets. Neither the presence of multiple world risk factors, regional risk factors, systematic currency risk factors, nor a country-specific currency subsumes the importance of the local market factor. All factors, including the local market, carry significant risk premia across a large proportion of countries. The contribution of pricing errors to total expected returns is large and time-varying.

JEL classification: C12, C13, C23, C51, C52 , G12, G15.

Keywords: approximate factor model, emerging markets, international asset pricing, large panel, market integration, time-varying risk premium.

---

*We thank Tony Berrada, Patrick Bolton, Andrea Buraschi, Pasquale Della Corte, Vihang Errunza, Thierry Foucault, Patrick Gagliardini, Harald Hau, Lukas Kremens (discussant), Sebastien Laurent, Thomas Maurer (discussant), Per Mykland, Elisa Ossola, Rogier Quaedvlieg, Corvino Raffaele (discussant), Ioanid Rosu, Paul Schneider (discussant), Christophe Spaenjers, Raman Uppal, Adrien Verdelhan, Dacheng Xiu, Paolo Zaffaroni (discussant), and seminar participants at the University of Chicago, Northwestern University, Boston University, HEC Paris, HEC Montreal, Imperial College Business School, Goethe University, University of Geneva, EFA 2019, FEC 2019, 2017 SFI Research Days, 2017 Big Data Workshop at PUC University, Computational and Financial Econometrics 2017 Conference, SGF 2018, Frontiers of Factor Investing 2018 Conference, TSE Financial Econometrics Conference, AFFI 2019, QFFE 2018, SoFiE 2018, FEBS 2019, 2019 Telfer Conference, HEC Liege, EDHEC, 2018 Annual Conference in International Finance, MFS 2018, EFMA 2018, ESEM 2018, European Commission JRC, ICSA, and $7^{th}$ Public Investors Conference for helpful comments. We are grateful for the Inquire Europe 2016 Grant. Langlois is grateful for financial support from the Investissements d'Avenir Labex (ANR-11-IDEX-0003/Labex Ecodec/ANR-11-LABX-0047). This paper was previously circulated with the title "Time-varying Risk Premia in Large International Equity Markets".

†University of Geneva and Swiss Finance Institute, UniMail, Bd. du Pont d'Arve 40, CH-1211 Geneva 4, Switzerland, ines.chaieb@unige.ch, +41 22 379 85 68.

‡HEC Paris, 1 rue de la Libération, Jouy-en-Josas 78350, France, langlois@hec.fr, +33 06 74 46 27 17.

§University of Geneva and Swiss Finance Institute, UniMail, Bd. du Pont d'Arve 40, CH-1211 Geneva 4, Switzerland, olivier.scaillet@unige.ch, +41 22 379 88 16.

# 1 Introduction

Understanding and measuring the determinants of expected returns in international equity markets is crucial to form optimal global investment portfolios, evaluate the performance of global equity managers, and obtain the cost of capital for global firms.

There are two steps required to test models of expected returns. The first step identifies the key factors that explain the cross-sectional variations of international stock returns. As recently emphasized in Pukthuanthong, Roll, and Subrahmanyam (2019) (p.1575): "Few topics in finance, arguably none, are more important than factor identification, because factors are the main principal determinants of investment performance and risk." The second step consists of estimating the factor risk premia and testing for the absence of arbitrage opportunities, namely measuring the magnitude of pricing errors.

Much of the previous work studying the factor structure and risk premia in international markets uses highly aggregated test assets, such as country portfolios, industry portfolios, or style portfolios. However, asset pricing tests and estimation of risk premia can differ when using aggregated test portfolios instead of individual stocks because of a potential aggregation bias.[1] This problem is particularly severe in a dynamic setting because aggregating assets into portfolios may induce misspecification in the dynamics of factor exposures.[2] Also, constructing sufficiently many characteristic-sorted portfolios to test candidate models is not possible in countries with a small number of stocks. Consequently, international asset pricing tests using portfolios are often carried on regions, which assumes a high degree of market integration of countries in a region.[3]

[1] Avramov and Chordia (2006) show that anomalies from conditional factor models differ a lot when considering single securities instead of portfolios. Ang, Liu, and Schwarz (2020) argue that we lose efficiency when only considering portfolios as base assets, instead of individual stocks, to estimate equity risk premia in models with time-invariant coefficients. Lewellen, Nagel, and Shanken (2010) advocate working with a large number of assets instead of a small number of portfolios exhibiting a tight factor structure.

[2] See Appendix H of the supplemental material of Gagliardini, Ossola, and Scaillet (2016) for theoretical arguments and empirical evidence for the U.S. market. Style-sorted portfolios result in stable betas which mask part of the time-variation in the risk premia.

[3] Previous international studies using style-portfolios as test assets run global- or regional-level tests to ensure a sufficiently high number of stocks in portfolios and enough dispersion in expected returns (see, for example, Fama and French, 1998, 2012, 2017). Studies at the country level use at most ten portfolios sorted on one characteristic or at most nine portfolios if they use a three-way size and book-to-market sort as the low number of stocks prevents a five-way sort as used in the U.S. market (see, for example, Fama and French, 1998; Hou et al., 2011; Griffin et al., 2003). Cattaneo, Crump, Farrell, and Schaumburg (2017) argue that an appropriate choice of the number of portfolios is key for drawing valid empirical conclusions and that we need more portfolios than currently considered in the literature.

In this paper, we propose a methodology tailored for large unbalanced panels of individual stock returns to identify the key risk factors in international equity markets and estimate their risk premia. We show that the local market is necessary to capture the factor structure in both developed markets (DM) and emerging markets (EM). Neither the presence of multiple world risk factors, regional risk factors, systematic currency risk factors, nor a country-specific currency subsumes the importance of the local market factor. Factor risk premia, including the local market risk premium, are significant in a high proportion of countries. Additionally, pricing errors are a large part of expected returns of individual stocks.

*Methodology preview.* We estimate at the individual stock level international factor models with time-varying factor exposures and risk premia using a large unbalanced panel of 64,392 stocks from 47 countries over the period 1985 to 2018.

We use an international arbitrage pricing theory (IAPT) in a multi-period economy (Hansen and Richard, 1987) with a flexible stochastic representation in which stock excess returns vary with their exposures to multiple risk factors. Our model accommodates cross-correlations in stock returns created by the conversion of local returns to a common numeraire currency, the U.S. dollar in our empirical application.

Our estimation method is based on two-pass regressions (Fama and MacBeth, 1973; Black et al., 1972) for individual stock returns and uses the bias correction of Gagliardini, Ossola, and Scaillet (2016) (GOS) to correct for the Error-in-Variable problem coming from the estimation of betas in the first pass regressions in large unbalanced panels. However, a direct application of the GOS methodology in an international setting is challenging because of the large number of parameters needed to model the time-variations in factor exposures and risk premia. Indeed, applying the GOS methodology off-the-shelf to an international setting results in few or even zero stocks kept for several countries (see Internet Appendix A). To address this issue, we provide an automatic selection procedure of the common and stock-specific instruments that capture the time variations in factor exposure for each stock without violating the no-arbitrage conditions.

Our objective is not to propose a new factor model, but to assess the ability of different combinations of leading factors, aggregated either at the country, regional, or world level, to explain

returns of individual international stocks. We consider market, size, value, and momentum factors as in Fama and French (2012), and also profitability and investment factors as in Fama and French (2017) and Hou, Xue, and Zhang (2015).[4] See, among others, Hou, Karolyi, and Kho (2011), Asness, Moskowitz, and Pedersen (2013), Titman, Wei, and Xie (2013) and Watanabe, Xu, Yao, and Yu (2013) for the role of size, value, momentum, profitability, and investment in international stock returns.

We build a set of factors for each of the 47 countries and then construct regional and world factors by aggregating country factors. We consider three regions of DMs (North America, Developed Europe, and Asia Pacific) and three regions of EMs (Latin America, Emerging Europe, Middle East and Africa, and Emerging Asia). Over our sample period, we show that market, size, value, momentum, profitability, and investment factors across regions deliver positive average returns and that higher average returns are related to higher volatility. For a textbook treatment of factor investing and investment issues in emerging markets, see Ang (2014) and Karolyi (2015), respectively.

*Main empirical results.* We first focus on determining which factors are required to explain the comovements between individual stock returns. Underlying our international no-arbitrage model is the assumption that idiosyncratic shocks are weakly cross-sectionally correlated as in the standard domestic version of the APT. Strong residual cross-correlations could be generated by a missing equity factor, a missing currency factor, or both. A missing currency factor may come from exchange rate shocks simultaneously affecting many stocks when local-currency returns are converted to U.S. dollar returns. Importantly, a remaining factor structure in residuals invalidates the estimation of risk premia and inference on asset pricing restrictions.

To assess the different factor models, we apply a new diagnostic criterion proposed in Gagliardini, Ossola, and Scaillet (2019a) (GOS2) that verifies if large unbalanced panel errors are weakly cross-sectionally correlated. We find omitted factors in the errors for models with world factors,

[4]The non-market factors are additional sources of systematic risk in International Arbitrage Pricing Theory (IAPT). See, for example, Liew and Vassalou (2000), Cooper and Priestley (2011), Cooper, Mitrache, and Priestley (2017), for evidence supporting risk-based interpretation of size and value, investment, and value and momentum, in the U.S. and international markets. But there exist alternative views to the risk-based explanation of the non-market factors (see, among others, Lakonishok, Shleifer, and Vishny, 1994). Kozak, Nagel, and Santosh (2018) argue that the success of a factor model does not discriminate between rational and behavioral explanations.

including those with profitability and investment factors, for both DMs and EMs. However, adding the excess country market factor—defined as the spread between the country market and the world market—is sufficient to capture the factor structure in the large cross-section of individual stock returns of almost all DMs and most EMs.[5] The excess country market factor is required even if we use regional factors instead of world factors. For example, a four-factor model with regional factors augmented with the excess country market factor captures the factor structure in stock returns for 100% of DMs and 83% of EMs. In contrast, the same model without the excess country market factor captures the factor structure of only 87% of DMs and 29% of EMs.

The importance of the excess country market factor is not explained by currency risk. For each country, we augment all candidate sets of factors with the country-specific currency returns. None of these models come close to the performance of models with the excess country market factor. In a second test, we augment each candidate set of factors with systematic currency factors: the carry factor of Lustig, Roussanov, and Verdelhan (2011) and the dollar factor of Verdelhan (2018).[6] These systematic currency factors are not sufficient to capture the part of the factor structure explained by the excess country market factor. Hence, our results show that the excess country market factor captures additional sources of risk beyond currency risk.

Our findings inferred from large panels of individual stock returns are important for several reasons. First, they contribute to the debate on the relative importance of global, regional, or country factors in international finance. In her review of this literature, Lewis (2011) states that (p.443): "returns depend upon more than a single factor and that at least some of these additional factors depend upon local sources of risk".[7] Bekaert, Hodrick, and Zhang (2009), Fama and French

[5]Past studies assess market segmentation by testing whether a country market factor orthogonal to the world market factor has explanatory power beyond the world market factor in time-series regressions, see, for example, Stehle (1977), Solnik (1974b), Errunza and Losq (1985). Errunza and Losq (1985) provide a theoretical foundation for using an orthogonalized country market factor. Karolyi and Wu (2018) find strong support for including a related factor in multi-factor models.

[6]Brusa, Ramadorai, and Verdelhan (2015) find an important role for dollar and carry currency risk factors in the cross-section of equity returns of style and country portfolios. Karolyi and Wu (2017) show that currency risk factors do not robustly contribute to capturing the time-series or cross-sectional variations in global and regional portfolio returns.

[7]Explicit barriers to investment such as those modeled in Errunza and Losq (1985) or differences in information across markets as modeled in Dumas, Lewis, and Osambela (2017) explain why returns are priced with global and local factors. See, also, Karolyi and Stulz (2003), for a review of the literature on whether assets are priced globally or locally.

(2012), and Fama and French (2017) find that regional factors perform better than global factors. However, our results show that regional factors are not sufficient to fully capture the factor structure, a necessary step prior to estimating risk premia.

Other studies find that models with multiple local factors perform better. For example, Griffin (2002) shows that the Fama-French three-factor model with country market, size, and value factors results in lower pricing errors than with factors aggregated at the global level. Similarly, Hou, Karolyi, and Kho (2011) find that a model with foreign and country market, momentum, and cash flow-to-price factors is not rejected.[8] Our results differ in that we need only the excess country market factor, not non-market local factors, to capture the factor structure in individual stock returns.

Identifying the factor structure in individual stocks also has implications for global portfolio management. Christoffersen, Errunza, Jacobs, and Langlois (2012) show that correlations between developed stock markets are high, especially in the latter part of their sample period, and therefore diversification benefits are low. High correlations suggest a predominance of common factors across developed *aggregate* stock markets. In contrast, our results based on *individual* stock returns underline the importance of the excess country market factor for explaining comovements. Therefore, country allocations continue to be an important consideration for global equity portfolio managers.

Having determined which sets of factors capture the factor structure in individual stock returns, we then estimate their risk premia. Our results indicate that factor risk premia are significantly positive in a large proportion of countries. In contrast, Jegadeesh et al. (2019) propose an alternative instrumental-variable estimators of the ex-post risk premia, and find that none of the market or non-market factors are associated with a significant risk premium in the cross-section of individual U.S. stock returns.

In particular, the risk premia of excess country market factors are significant in 43% to 84% of countries depending on the model and region. Unsurprisingly, their magnitude is larger in EMs than in DMs.

The significant premium for the excess country market factor in world and regional models

[8] See also Stehle (1977), Korajczyk and Viallet (1989), Rouwenhorst (1999), and Eun, Lai, de Roon, and Zhang (2010).

not only for EMs but also for many DMs provides evidence of segmentation along both world and regional lines. Indeed, the necessary and sufficient condition for market integration is that risk premia on a common set of risk factors that drive returns in all countries are equal and each country-specific factor has a zero risk premium.[9]

We document how pricing errors contribute to the total expected return of equal-weighted portfolios in each region. The time-series median absolute pricing error, standardized by the total expected return, range from 23% to 63% across models and regions and exhibit large time-variations. Finally, we also document how time-variations in factor exposures are primarily driven by stock-specific characteristics.

Our results are important because most empirical studies of international equity markets have so far focused on risk premia estimated from test portfolios. This approach hinders the estimation of the risk premium dynamics since the nature of their construction targets stable factors loadings (see GOS for a test and further discussion). To the best of our knowledge, this is the first paper that document time-varying risk premia and pricing errors from multi-factor models estimated from such a large panel of individual international stock returns.

Data quality is especially important when testing asset pricing models at the individual stock level. Here we do not rely on value-weighted test portfolios in which aggregation attenuates data errors frequently found in international stock databases. We conduct a comprehensive comparative analysis of international stock databases by comparing individual stock data in each country using data from Thomson Reuters Datastream and S&P Compustat Global. In Internet Appendix B, we build an exhaustive list of filters and data corrections to use with the S&P Compustat Global database that supplement those used in the literature.

*Related literature.* Our paper contributes both to the theoretical and empirical literature on IAPT. On the theoretical side, Solnik (1983) shows how to generalize the APT to an international setting for time-invariant models if exchange rates are explained by the same factor structure as stock returns and idiosyncratic shocks are cross-sectionally independent. Ikeda (1991) does not assume that exchange risk is spanned by the factors, and shows how to derive the no-arbitrage

[9] Ideally, we would like to estimate models in which are included all country market factors. The curse of dimensionality precludes the estimation of such models.

restriction by hedging exchange rate risk in continuous time.[10] Our model is set in discrete time and uses a flexible factor structure with weak cross-sectional dependence in the residuals potentially induced by currency conversion of stock returns.

Our work is also part of a growing literature on asset pricing tests using individual stocks. Jegadeesh et al. (2019) propose an instrumental variable estimator of the ex-post risk premia, as in Shanken (1992b), when the time series dimension $T$ is small. Their estimator controls for the Error-in-Variable problem in asset pricing tests with individual stocks. Alternatively, we build on the GOS methodology that estimate and correct for the Error-in-Variable bias in an unbalanced panel of individual stock returns when the time series dimension $T$ diverges to exploit a law of large numbers for estimating and a central limit theorem for testing, see Gagliardini, Ossola, and Scaillet (2019b) for a review of this approach. Raponi, Robotti, and Zaffaroni (2020) and Kim and Skoulakis (2018) also propose methodologies for testing factor models when the cross-section of stocks is large, but the time-series sample size is small. We instead rely on the double asymptotic theory in GOS in which both the cross-sectional and the time-series dimensions are large.

Pukthuanthong, Roll, and Subrahmanyam (2019) propose a protocol for factor identification based on the correlations between a set of candidate factors and principal components in returns. We instead use the diagnostic criterion of Gagliardini, Ossola, and Scaillet (2019a) built on the same theoretical setup as the asset pricing testing methodology we use. This approach ensures we work under the same theoretical umbrella; specifically, an IAPT suited to large dimensional conditional factor models. Alternatively, Giglio and Xiu (2019) propose a method for estimating factor risk premia when some factors are omitted and Uppal, Zaffaroni, and Zviadadze (2019) show how to correct for missing factors when estimating the APT. Our approach differs in that we first make sure that all relevant factors have been specified for an approximate time-varying factor structure for large unbalanced panels of individual stock returns (i.e., no omitted factors remain in the model errors and we do not suffer from an omitted-variable bias) before estimating factor risk premia.

Beyond all of these methodological differences, the main distinction between these papers and

[10] Exchange risk hedging by holding locally riskless bonds is also proposed in utility-based International Asset Pricing Models (IAPM). See, for example, Solnik (1974a), Sercu (1980), Stulz (1981), and Adler and Dumas (1983). Among others, Dumas and Solnik (1995), De Santis and Gerard (1998), Brusa, Ramadorai, and Verdelhan (2015), and Karolyi and Wu (2017) test conditional international asset pricing models with currency risk.

ours is that they focus on the U.S. stock market, whereas we test factor models on international equity markets. Therefore, we provide new empirical results on the factor structure and the significance of time-varying risk premia in international equity markets.

*Layout.* We present the theoretical model in Section 2, provide our empirical methodology in Section 3, describe our data in Section 4, and present our empirical results in Section 5, and Section 6 concludes. Appendix A details the key inference tools used to get our results. We report details on the construction of the equity database and additional estimation results in the Internet Appendix.

# 2 A multi-period international APT

In this section, we provide a multi-period IAPT with currency risk and a varying degree of market segmentation. We start by describing the factor structure for excess returns. We then combine this factor structure with the absence of arbitrage opportunities to obtain asset pricing restrictions. We work in a multi-period economy under an approximate factor structure with a continuum of assets as in GOS, and refer to their proof for asset pricing results as well as a detailed discussion on the reasons for using a continuum.

## 2.1 A time-varying factor model for stock returns with currency risk

We consider $C$ countries. In each country $c \in \{1, ..., C\}$, we use the index $\gamma \in [0, 1]$ to designate an asset belonging to a continuum of assets on an interval normalized to $[0, 1]$ without loss of generality. We assume that each country has its own currency and we use the U.S. dollar (USD) as the numeraire currency. The return in USD at time $t$ on asset $\gamma$ in country $c$ in excess of the U.S. risk-free rate, $r_{c,t}(\gamma)$, follows the factor structure:

$$r_{c,t}(\gamma) = a_{c,t}(\gamma) + b_{c,t}(\gamma)' f_{c,t} + \varepsilon_{c,t}(\gamma). \tag{1}$$

In Equation (1), $a_{c,t}(\gamma)$ and $b_{c,t}(\gamma)$ are a time-varying intercept and time-varying exposures to $K$ systematic factors $f_{c,t}$.

Both the intercept $a_{c,t}(\gamma)$ and factor loadings $b_{c,t}(\gamma)$ are $\mathcal{F}_{t-1}$-measurable, where $\mathcal{F}_{t-1}$ is the information available to all investors at time $t-1$.[11] The error terms have mean zero, $E[\varepsilon_{c,t}(\gamma)|\mathcal{F}_{t-1}] = 0$, and are uncorrelated with the factors, $Cov[\varepsilon_{c,t}(\gamma), f_{c,t}|\mathcal{F}_{t-1}] = 0$. These conditions allow to identify $a_{c,t}(\gamma)$ and $b_{c,t}(\gamma)$ as time-varying regression coefficients.

Our factor model in Equation (1) applies to international asset returns converted to a common currency. Under this assumption, the factor model also applies to foreign risk-free bonds which are risky assets when measured in USD. Hence, we implicitly assume that currency returns follow the same factor structure,

$$r_{c,t}(\gamma_s) = a_{c,t}(\gamma_s) + b_{c,t}(\gamma_s)' f_{c,t} + \varepsilon_{c,t}(\gamma_s), \tag{2}$$

where $r_{c,t}(\gamma_s)$ is the excess return on country $c$ currency $\gamma_s$ in units of the numeraire currency (USD).[12]

Our factor structure (1)-(2) is similar to Solnik (1983) in that we impose a factor structure on returns converted to a numeraire currency. However, our model differs from his in two important aspects. First, Solnik (1983) uses a common set of factors for all international stocks, which is appropriate for the case of integrated markets. In contrast, our set of systematic factors $f_{c,t}$ can be specific to country $c$ or common across several countries. We further discuss the issue of market integration in Section 2.2 below. Second, Solnik (1983) assumes that idiosyncratic shocks $\varepsilon_{c,t}(\gamma)$ are cross-sectionally independent. In our model, we explicitly consider the impact of currency conversion on correlations across stocks since we do not impose a priori an exact factor structure in which errors have a diagonal covariance matrix.

There are two ways in which currency risk may impact the correlation structure of USD-denominated stock returns. First, stock returns in one currency converted to USD are all impacted by the currency-specific shock $\varepsilon_{c,t}(\gamma_s)$, which may result in higher cross-correlation for securities in this country. Second, currency-specific shocks $\varepsilon_{c,t}(\gamma_s)$ can be correlated across currencies if there exists currency-specific factors (i.e., $\varepsilon_{c,t}(\gamma_s)$ follows a factor structure). In such a case, this

[11] GOS impose some non-degeneracy conditions on $a_{c,t}(\gamma)$ and $b_{c,t}(\gamma)$, which prevent that only a few assets load on a factor. Other regularity conditions are assumed for the theory and inference procedures to work (see GOS for details).

[12] In Equation (2), the terms on the right-hand side absorb the terms involving the risk-free rate of country $c$.

currency-specific factor structure results in higher correlations between countries. In both cases, neglecting the correlations that currency conversion may induce within and across blocks of securities invalidates inference on risk premia. There can also be other sources of correlation between stock returns besides currencies, such as industry effects.

To handle potential correlations across idiosyncratic shocks, we impose an approximate factor structure (as opposed to an exact factor structure) for model (1) in each country $c$. Precisely, for any sequence $(\gamma_{i,c})$ in $[0,1]$, for $i = 1, ..., n_c$, let $\Sigma_{\varepsilon_c,t,n_c}$ denote the $n_c \times n_c$ conditional variance-covariance matrix of the error vector $[\varepsilon_{c,t}(\gamma_{1,c}), ..., \varepsilon_{c,t}(\gamma_{n_c,c})]'$ conditional on $\mathcal{F}_{t-1}$. We assume that there exists a set such that the ratio of the largest eigenvalue of $\Sigma_{\varepsilon_c,t,n_c}$ to $n_c$ converges to 0 in $L^2$ as $n_c$ grows. The validity of this assumption is also sufficient if we want to estimate risk premia for an integrated world market with all countries or for an integrated region with all countries in that region.[13]

Chamberlain and Rothschild (1983) (CR) use a sequence of variance-covariance matrices for the error terms that have uniformly bounded eigenvalues. The GOS and our theoretical settings do not assume that the largest eigenvalue is asymptotically bounded. Instead, the assumption is on the growth rate of the largest eigenvalue (see Assumption APR3 in GOS, p. 992). Therefore, the largest eigenvalue can be unbounded as long as it does not grow too fast relative to the number of assets $n$. This assumption relaxes the strong assumption of CR and generalizes previous international APTs to a more flexible and realistic market structure. In particular, we allow for block cross-correlations between idiosyncratic shocks that may be induced by currency conversion. Such a structure may violate the CR assumption, for example when blocks become larger and more numerous, but is compatible with our assumption (see Appendix F of GOS). In Section 5.1, we empirically examine which set of candidate risk factors captures the factor structure in excess stock returns denominated in USD. The weak cross-sectional dependence in the errors implies that no systematic equity or currency factor is missing.

[13] Our assumption on the covariance matrix of idiosyncratic shocks in country $c$ translates into a similar result for the covariance matrix of all idiosyncratic shocks across the $C$ countries. Indeed, the largest eigenvalue of a positive semi-definite matrix is less than or equal to the sum of the largest eigenvalue associated to each diagonal block. This is true without the need for the off-diagonal blocks to be zeros, that is, without assuming zero correlation between countries. Hence, the largest eigenvalue of the conditional covariance matrix of all idiosyncratic shocks $\varepsilon_{c,t}$ divided by the total number of stocks $n = \sum_{c=1}^{C} n_c$ also converges to 0 in $L^2$ as $n$ grows.

## 2.2 Asset pricing restrictions

We now combine the approximate factor structure introduced in the last section with the absence of asymptotic arbitrage opportunities to obtain asset pricing restrictions.

If there are no asymptotic arbitrage opportunities, there exists a unique $\mathcal{F}_{t-1}$-measurable $K$-by-1 vector $\nu_{c,t}$ such that

$$a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}, \tag{3}$$

for almost all $\gamma \in [0,1]$ in country $c$. If there does not exist such a vector $\nu_{c,t}$, there are arbitrage opportunities in country $c$. Equivalently, we can rewrite the asset pricing restriction (3) as the usual linear relation between conditional expected excess returns and conditional factor risk premia:

$$E\left[r_{c,t}(\gamma)|\mathcal{F}_{t-1}\right] = b_{c,t}(\gamma)'\lambda_{c,t}, \tag{4}$$

where $\lambda_{c,t} = \nu_{c,t} + E\left[f_{c,t}|\mathcal{F}_{t-1}\right]$ is the vector of the conditional factor risk premia in country $c$.

In the CAPM, we have $K = 1$ and $\nu_{c,t} = 0$. More generally, in a multifactor model in which factors are excess returns on tradable portfolios, we have $\nu_{c,t,k} = 0$, for $k = 1, ..., K$. In the empirical section below, we use as factors long-short portfolios built to capture the size, value, momentum, profitability, and investment effects. As these factors imply buying and short-selling a large amount of securities and their returns do not reflect transaction and short-selling costs, they may not be tradable, especially in less developed markets.[14] A non-zero $\nu_c$ may suggest large trading costs, and differences in those trading costs contribute to market segmentation. Therefore, we test both the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$, and the asset pricing restrictions with tradable factors, $a_{c,t}(\gamma) = 0$. The latter corresponds to the usual testing procedure of Gibbons, Ross, and Shanken (1989) but in a large unbalanced panel structure.

We are not affected by the Shanken (1982) critique, namely the problem that we cannot test empirically the finiteness of the sum of squared pricing errors for a given countable economy,

[14] Using spanning tests, Bekaert and Urias (1996) and De Roon, Nijman, and Werker (2001) show that diversification benefits of emerging markets disappear when accounting for short-selling constraints and transaction costs in those markets.

$\sum_{i=1}^{\infty}\left(a_{c,t}(\gamma_{i,c}) - b_{c,t}(\gamma_{i,c})'\nu_{c,t}\right)^2$; see also the discussion in Shanken (1992a). Working with a continuum of assets, GOS show that if the asset pricing restriction (3) holds, then this sum over a countable set of assets is necessarily finite. If, on the other hand, the asset pricing restriction (3) does not hold, the sum is necessarily infinite.[15] Following the empirical methodology developed in GOS which we extend in Section 3, the restriction (3) is testable with large equity datasets and large sample sizes. Therefore, working with a continuum of assets as in GOS, instead of a countable set of assets as in CR, circumvents the methodological difficulty, raised by Shanken (1982), of directly testing the asset pricing restrictions of CR.

Finally, we distinguish between the importance of a country-level factor and market segmentation. A country-level factor, say the German stock market, may drive German stock returns through Equation (1), but this is not a sufficient condition for market segmentation. Nor is a common set of factors $f_t$ driving stock returns in Switzerland and Germany a sufficient condition for these two markets to be integrated. Indeed, "we cannot infer capital market integration from factor structure or correlation structure", as stated by Cho, Eun, and Senbet (1986).[16] The necessary and sufficient condition for market integration is that each country-level factor has a zero risk premium and risk premia for a common set of risk factors that drive returns in both countries are equal.

In our empirical tests, we use the asset pricing restrictions (3) to test different factor model specifications. We investigate models with a country-specific set of risk factors, $f_{c,t}$, and models with a common set of factors, $f_t$, across countries.

# 3 Empirical methodology

This section describes our empirical methodology. We first discuss how we parameterize time-varying factor exposures and risk premia. Second, we explain our choice of instruments. Next, we describe the two-pass approach. Finally, we describe the diagnostic criterion for the factor structure and the test statistic for the asset pricing restrictions.

[15] See p. 994 and Appendix E of GOS for a detailed discussion.

[16] Cho, Eun, and Senbet (1986), Heston, Rouwenhorst, and Wessels (1995), and Sentana (2002) test market integration by testing equality of factor risk premia across countries.

### 3.1 Specification of the time-varying factor exposures and risk premia

The conditioning information $\mathcal{F}_{t-1}$ contains $Z_{c,t-1}$ and $Z_{c,t-1}(\gamma), c = 1, ..., C$. The $p$-by-1 vector of lagged instruments $Z_{c,t-1}$ is common to all stocks of country $c$ and may include a constant, past observations of the factors, and some additional variables such as macroeconomic and financial variables common to all countries or country-specific. The $q$-by-1 vector of lagged instruments $Z_{c,t-1}(\gamma)$ is specific to stock $\gamma$ in country $c$, and may include past observations of firm characteristics.

To obtain a workable version of the conditional IAPT from the previous section, we use linear specifications. First, the vector of factor loadings is a linear function of lagged instruments $Z_{c,t-1}$ (see, for example, Shanken, 1990; Ferson and Harvey, 1991; Dumas and Solnik, 1995) and $Z_{c,t-1}(\gamma)$ (see, for example, Avramov and Chordia, 2006),

$$b_{c,t}(\gamma) = B_c(\gamma) Z_{c,t-1} + C_c(\gamma) Z_{c,t-1}(\gamma), \tag{5}$$

where $B_c(\gamma)$ is a $K$-by-$p$ matrix and $C_c(\gamma)$ is a $K$-by-$q$ matrix.

Second, the vector of risk premia is a linear function of lagged instruments $Z_{c,t-1}$ (see, for example, Dumas and Solnik, 1995; Cochrane, 1996; Jagannathan and Wang, 1996),

$$\lambda_{c,t} = \Lambda_c Z_{c,t-1}, \tag{6}$$

where $\Lambda_c$ is a $K$-by-$p$ matrix. Finally, the conditional expectation of the factors $f_{c,t}$ given the information $\mathcal{F}_{t-1}$ is,

$$E\left[f_{c,t} | \mathcal{F}_{t-1}\right] = F_c Z_{c,t-1} \tag{7}$$

where $F_c$ is a $K$-by-$p$ matrix.

### 3.2 Choice of instruments

We use the lagged world dividend yield, $DY_{t-1}$, and a country lagged dividend yield, $DY_{c,t-1}$, as common instruments in addition to a constant. Hence, $Z_{c,t-1} = (1, DY_{t-1}, DY_{c,t-1})'$ in our

empirical application.[17] To ensure that conditional expectations of world factors in Equation (7) are equal across countries, we set the elements of $F_c$ corresponding to their loading on the country dividend yield to zero. In the interest of parsimony, we impose that both factor loadings $b_{c,t}(\gamma)$ in Equation (5) and the vector $\nu_{c,t}$ in Equation (3) do not load on the global instrument $DY_{t-1}$.[18] In models where we use regional instead of world factors, $DY_{t-1}$ is the regional dividend yield. The country, world, and regional dividend yields are standardized to have a mean of zero and a standard deviation of one.

In asset pricing tests with test portfolios, it is the *composition* of the test assets that varies over time as stocks with similar characteristics are assembled into different portfolios. In such a case, we can expect that estimating betas using either the full sample or rolling windows would adequately capture each test portfolio factor loadings. For example, a size sorted portfolio with small capitalization firms will consistently have a positive loading on a size factor over time.

However, when we test asset pricing models using individual stocks, the composition of the test assets is fixed (i.e., one stock), and it is their *characteristics* that vary over time. As a firm evolves and its stock characteristics change over time, we cannot expect its betas to be constant. Consequently, estimating betas over rolling windows would necessarily lag its true time-varying factor exposures as time-invariant OLS estimates average recent and more distant exposures within the rolling window.

Therefore, as stock-specific instruments, we use the cross-sectional ranks (within each country) of the size, value, momentum, profitability, and investment characteristics depending on which factors are included in the model.[19] This method, which differs from the empirical strategy of GOS, has several advantages. First, we directly capture the time-varying exposure of single stocks

[17] Several studies show that the dividend yield help predict time variation in international equity returns and use it as an instrumental variable (see, for example, Harvey, 1991; Ferson and Harvey, 1991, 1993). Other variables such as the term spread and the default spread have also some predictive power for equity returns and are used as instruments in past studies.

[18] The vector of conditional risk premia involves both the conditional expectation of the factors, via the coefficients matrix $F_c$, and the process $\nu_{c,t}$. The restriction on $\nu_{c,t}$ is equivalent to restricting the element of $F_c$ for the loading of the global factor on the global instrument to be equal to its corresponding element in $\Lambda_c$ in Equation (6). Therefore, we assume that the global factor risk premium depends on $DY_{t-1}$ only through its conditional expectation.

[19] The only exception is the CAPM for which we use the size and value characteristics. Connor, Hagmann, and Linton (2012) also use the corresponding firm characteristic to model the beta of a given factor, that is, the firm size for the beta of the size factor, etc. Among others, Shanken (1990), Avramov and Chordia (2006), and GOS assume factor loading that vary with firm characteristics.

to factors. For example, if the market capitalization of a stock suddenly decreases, it becomes part of the long leg of the size factor and its cross-sectional size rank decreases. This instrument will accordingly capture its increased exposure to the size factor. Another advantage of using the cross-sectional ranks of characteristics instead of their values is that it attenuates the impact of potential data errors in individual stock characteristics. See Freyberger, Neuhierl, and Weber (2018), Asness, Frazzini, and Pedersen (2019), Kozak, Nagel, and Santosh (2020), Kelly, Pruitt, and Su (2019), and Kim, Korajczyk, and Neuhierl (2019) for a similar choice.

### 3.3 Two-pass cross-sectional regression approach

In this section, we summarize the two-pass approach used to estimate factor risk premia and the extension of the GOS methodology to accommodate our international setting. All details are provided in Appendix A, including clarifications on the differences with the GOS methodology. For simplicity, we use in this section and the Appendix the subscript $i$ to denote a stock instead of the index $\gamma$.

Our specification choices for factor exposures and factor risk premia (Equations (5), (6), and (7)) combined with the asset pricing restrictions, $a_{i,c,t} = b_{i,c,t}\nu_{c,t}$, imply that a stock intercept is a function of lagged instruments, namely,

$$a_{i,c,t} = Z'_{c,t-1}B'_{i,c}\left(\Lambda_c - F_c\right)Z_{c,t-1} + Z'_{i,c,t-1}C'_{i,c}\left(\Lambda_c - F_c\right)Z_{c,t-1}. \tag{8}$$

The no-arbitrage condition in Equation (8) shows that, with time-varying factor exposures and risk premia, a stock intercept is a quadratic form of instruments, not a linear form.

The first pass consists in running the time-series regressions,

$$r_{i,c,t} = \beta'_{i,c}x_{i,c,t} + \varepsilon_{i,c,t}, \tag{9}$$

with $x_{i,c,t} = \left(x'_{i,c,t,1}, x'_{i,c,t,2}\right)'$ and $\beta_{i,c} = \left(\beta'_{i,c,1}, \beta'_{i,c,2}\right)'$. In this equation, the first group of regressors, $x_{i,c,t,1}$, combines all terms for the intercept in Equation (8), the second group of regressors, $x_{i,c,t,2}$, combines factors scaled by common instruments, $Z_{c,t-1}$, and stock-specific instruments,

$Z_{i,c,t-1}$, and the $\beta_{i,c,1}$ and $\beta_{i,c,2}$ are their respective coefficients. The first group in $x_{i,c,t,2}$ takes the interpretation of scaled factors (Cochrane, 2005), while not the latter since they depend on $i$. We provide the definitions for all terms in Appendix A.

We cannot directly apply the GOS methodology because their trimming conditions to run the first-pass time-series regressions (9) result in few or even zero stocks kept for several countries (see Internet Appendix A).[20] Precisely, we show in Section 5.1 that multiple factors are required to capture the factor structure in individual international stock returns. Further, we find in Section 5.4 that factor exposures are related to several instruments. Therefore, the proliferation of parameters to estimate, coupled with the limited number of stocks in some countries, means that too few stocks have a time-series long enough and regressors in the time-series regression well-conditioned enough to satisfy the trimming conditions.

To handle the large number of parameters needed to capture time-varying factor exposures in multi-factor models, we extend the GOS methodology by iteratively selecting for each stock the most important instruments $Z_{c,t-1}$ and $Z_{i,c,t-1}$. Naively removing regressors in $x_{i,c,t,2}$ could introduce arbitrage if we do not select the appropriate regressors in $x_{i,c,t,1}$. Our approach ensures that the chosen model specification is consistent with no-arbitrage conditions.

We show in Appendix A how the GOS estimation methodology and asset pricing tests are modified to account for stock-specific choices of instruments provided by our data-driven sequential approach. GOS use the same set of instruments across all U.S. stocks. In our international setting, we allow for a different set of instruments for each stock, and the number of selected stock-specific instruments varies across stocks. This necessary flexibility imposes a few adjustments for the validity of the inference. We show in Internet Appendix A that our new methodology yields a sufficient number of stocks to estimate multi-factor models, in contrast to the GOS methodology.

The second pass consists in computing a cross-sectional weighted least-squares estimator of the factor risk premia $\nu_c$ by regressing the $\hat{\beta}_{i,c,1}$s on the $\hat{\beta}_{i,c,3}$s as, $\beta_{i,c,1} = \beta_{i,c,3}\nu_c$, where $\beta_{i,c,3}$ is a

[20] GOS remove a stock if its time-series of $T_{i,c}$ observations is shorter than 60 months or the condition number of the matrix $\hat{Q}_{x,i,c} = \frac{1}{T_{i,c}} \sum_t x_{i,c,t} x'_{i,c,t}$ underlying the OLS estimate of $\beta_{i,c}$ is above 15. When we work with a large set of instruments and factors, the dimension of $x_{i,c,t}$ in the GOS methodology quickly becomes large resulting in an ill-conditioned matrix $\hat{Q}_{x,i,c}$ and numerically unstable OLS estimates (Belsley et al., 2004; Greene, 2008).

transformation of the coefficients $\hat{\beta}_{i,c,2}$ and $\nu_c$ is the vectorized form of $\Lambda_c - F_c$ given in Equations (A.5) and (A.6), respectively.

To obtain estimates of the time-varying risk premia, $\hat{\lambda}_{c,t}$, we first obtain estimates of $F_c$ by a SUR regression of factors $f_{c,t}$ on lagged common instruments $Z_{c,t-1}$. Then, we obtain $\hat{\Lambda}_c$ through the relation $\nu_c = \text{vec}\,(\Lambda_c' - F_c')$ and $\hat{\lambda}_{c,t} = \hat{\Lambda}_c Z_{c,t-1}$.

### 3.4 Estimation and tests

Our estimation method is based on two-pass regressions for individual stock returns as detailed in Section 3.3 and applies the bias correction of GOS to correct for the Error-in-Variable problem coming from the estimation of betas in the first-pass regressions in large unbalanced panels (see Appendix A).

To empirically assess whether a factor model successfully captures systematic risk, we use the diagnostic tool of GOS2 that checks whether there remains a common factor structure in idiosyncratic shocks $\varepsilon_{c,t}(\gamma)$.[21] The diagnosis works as follows. If there are no factors in the residuals, the maximum eigenvalue of the scaled matrix of the residuals goes to zero at a faster rate than a penalty term as $n_c$ and $T_c$ increase. On the contrary, if there remains at least one factor in the residuals, then the maximum eigenvalue stays large and positive. Hence, GOS2 use a negative value of the diagnostic criterion, given in Equation (A.10), to conclude that there does not remain any factor structure in the residuals and that we achieve weak cross-sectional dependence for the errors.

The diagnostic criterion makes the right model selection decision if its value is negative when there are no factors in the residuals, or its value is positive when there is at least one remaining factor. As the sample size ($T_c$) and the number of stocks ($n_c$) go to infinity, the probability that the diagnostic criterion makes the right decision goes to 1. Therefore, the diagnostic criterion is not a statistical test as it has a zero probability of wrongfully rejecting the null hypothesis that the set of factors is correctly specified (i.e., a Type I error).

However, when the sample size ($T_c$) and the number of stocks ($n_c$) are small, it is natural to

[21] GOS2 extend the well-known procedures of Bai and Ng (2002) and Bai and Ng (2006) to unbalanced panels and estimated errors instead of the true ones. See also Onatski (2010) and Ahn and Horenstein (2013).

wonder how reliable is the diagnostic criterion. In an extensive Monte Carlo simulation with sample sizes similar to those in our empirical analysis, we find that the probability of making the right model selection decision is higher than 99%, even when the number of stocks is as small as 150 (see Internet Appendix C).

Finally, we evaluate the asset pricing restrictions by computing the weighted sum of squared residuals (SSR) of the second-pass cross-sectional regression $Q_e = E\left[e_{c,t}(\gamma)e'_{c,t}(\gamma)\right]$ where $e_{c,t}(\gamma) = a_{c,t}(\gamma) - b_{c,t}(\gamma)'\nu_{c,t}$. Under the null hypothesis that the asset pricing restrictions hold, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$, the test statistic is normally distributed with expected value and variance given in Appendix A.7. We can test the asset pricing restrictions with tradable factors, $a_{c,t}(\gamma) = 0$, by replacing $\nu_{c,t}$ by a vector of 0 in these expressions. The latter yields a test similar to the well-known test developed by Gibbons, Ross, and Shanken (1989) but altered here to suit our large unbalanced panel instead of a small $n$.[22]

# 4 Data

Our empirical analyses require individual stock returns and characteristics. We start by describing our data construction for individual stock returns and then describe how we construct risk factors.

## 4.1 Individual stock data

We use data for 64,392 stocks from 23 DMs and 24 EMs. Our data are monthly, denominated in U.S. dollars, in excess of the U.S. one-month T-Bill rate, and cover the period January 1985 to October 2018.

We conduct a comprehensive comparative analysis of different global stock databases. Fama and French (2012, 2017) use data from Bloomberg, which they complement with Datastream, to obtain stock returns and accounting variables for 23 DMs. Asness, Moskowitz, and Pedersen (2013) use S&P Compustat Global (xpressFeed) to obtain stock returns and accounting variables for the same

[22]See also Shanken (1985) for a test based on cross-sectional regression residuals and characterization of the small-sample properties of the cross-sectional GLS estimates.

23 DMs. In contrast, Hou, Karolyi, and Kho (2011), Karolyi and Wu (2018), Lee (2011), and De Moor and Sercu (2013) use data from Datastream, on which they apply different filters to handle data errors. In Internet Appendix B, we detail our construction methodology and comparative analysis of data coming from Compustat and Datastream and provide an exhaustive list of filters and data corrections. For reasons listed in Internet Appendix B, we use data from Compustat in this paper.

We retrieve all securities classified as common or ordinary shares, but keep only stocks listed on a country's major stock exchange. We define a major stock exchange as the one with the highest number of listed equities. However, we include more than one stock exchange in some countries: Brazil (Rio de Janeiro and Bovespa), Canada (Toronto and TSX Venture), China (Shanghai and Shenzhen), France (Paris and NYSE Euronext), Germany (Deutsche Boerse and Xetra), India (BSE and National Stock Exchange), Japan (Tokyo and Osaka), Russia (Moscow and MICEX), South Korea (Korea and KOSDAQ), Switzerland (Swiss Exchange and Zurich), United Arab Emirates (Abu Dhabi and Dubai), and the U.S. (NYSE, NYSE Arca, Amex, and NASDAQ).

We keep only countries with at least a 10-year continuous period. We combine the 23 DMs into three regions: (i) North America (Canada and U.S.); (ii) Developed Europe (Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and United Kingdom); and (iii) Asia Pacific (Australia, Hong Kong, Japan, New Zealand, and Singapore). We combine the 24 EMs into three regions: (i) Latin America (Argentina, Brazil, Chile, Mexico, and Peru), (ii) Emerging Europe, Middle-East and Africa (Jordan, Morocco, Oman, Poland, Russia, Saudi Arabia, South Africa, Turkey, and United Arab Emirates), and (iii) Emerging Asia (China, India, Indonesia, Malaysia, Pakistan, Philippines, South Korea, Sri Lanka, Taiwan, and Thailand).

Table 1 provides summary statistics for our data. We provide summary statistics for countries in Panel A, the minima, averages, and maxima across countries in Panel B, and summary statistics for regions in Panel C. Except for the North American region, the return data for DMs usually start in the late 1980s. The EM return data start in the mid-1990s. The number of stocks varies over time and across countries. As of October 2018, there are 41,519 stocks in DMs and 22,070 in EMs.

The minimum (maximum) number of stocks in a country is 97 (11,776) for Morocco (U.S.), the minimum (maximum) number of stocks in a region is 1,109 (17,481) in Latin America (Emerging Asia).

### 4.2 Risk factors

We consider six *types* of factors. We first use the excess return on the value-weighted market portfolio of all stocks as in the CAPM. Next, we use size, value, and momentum factors from the Fama-French-Carhart four-factor model. Finally, we consider profitability and investment factors used in the five-factor model of Fama and French (2015) and the $q$-factor model of Hou, Xue, and Zhang (2015).[23] Feng, Giglio, and Xiu (2019) show that profitability and investment factors have significant explanatory power for expected returns of U.S. stocks.

We use the market capitalization of a stock to measure size, and the monthly updated book-to-price ratio constructed as in Asness and Frazzini (2013) to measure value. See Barillas and Shanken (2018) for recent evidence on the importance of using the monthly updated value factor.

Each month and for each country, we use all stocks with a valid market capitalization at the end of the previous month as test assets and to construct a value-weighted market factor. We use the subset of these stocks with a valid book-to-price ratio to construct the size and value factors. All stocks in a country are assigned to two size groups using the median market capitalization for U.S. stocks and the $80^{th}$ market capitalization percentile for non-U.S. stocks. Within each size group, we separate stocks into three value-weighted portfolios based on the $30^{th}$ and $70^{th}$ percentiles of the book-to-price ratios. We follow Asness, Moskowitz, and Pedersen (2013) and use conditional sorts to ensure a balanced number of stocks in each portfolio. We require at least 20 available stocks to compute a factor return.

The size factor is the average of the return of the three small-size portfolios minus the average return on the three large-size portfolios. The value factor is the average return of the small and large portfolios with high book-to-price ratios minus the average return of the small and large portfolios

[23] Our profitability factor differs from theirs as we use a double sort on size and cash profitability and they use a triple sort on size, ROE, and investment to build the profitability factor. The available number of stocks in some countries makes it difficult to do a triple sort. We rely on double sorts for all factors to keep the same construction methodology across countries.

with low book-to-price ratios. The momentum, profitability, and investment factors are constructed similarly as the value factor replacing book-to-price ratios, respectively, by each characteristic in the second ranking. To measure momentum, we use the past 12-month cumulative return on the stock and skip the most recent month return (see, for example, Jegadeesh and Titman, 1993). To measure profitability, we use cash profitability, which is gross profitability unaffected by accruals, divided by book value of equity.[24] We use the relative change in total asset values to measure investment (see Fama and French, 2017; Hou, Xue, and Zhang, 2015).

We form one set of factors in each country and compute aggregate factors for each region and at the world level by value-weighting country-specific factors using lagged country total market capitalizations denominated in USD.

Table 1 reports the annualized average returns in columns (iii) to (viii) and volatilities in columns (ix) to (xiv) of the market excess return, size, value, momentum, profitability, and investment factors for each country, as well as at the world level and by region.

The annualized mean returns averaged across countries of these six factors are 8.25%, 1.87%, 4.36%, 9.80%, 3.16%, and 2.52%, respectively. The average market excess return is positive for all markets except Saudi Arabia. Factor average returns are positive for 65.96% of countries for size, 87.23% for value, 95.74% for momentum, 78.72% for profitability, and 76.60% for investment (unreported proportions). However, there are substantial cross-country differences in the magnitude of these historical average returns. Annualized factor volatilities range from 5.42% to 38.27%. Higher factor average return is associated with higher risk; the correlation between average returns and volatilities across all countries and factors is 0.35 (unreported).

Panel C of Table 1 presents the average returns and volatilities for the factors aggregated by DMs, EMs, and region. All average factor returns for DMs and EMs are positive. The EM market factor has higher volatility than the DM market factor, a well-documented fact in the literature. However, size, value, and momentum factors in EMs deliver lower volatility than their respective DM factors. The risk/return profiles of size, profitability, and investment are similar across DMs

[24] Ball, Gerakos, Linnainmaa, and Nikolaev (2016) and Fama and French (2018) show that a cash profitability factor outperforms a gross-profitability factor in explaining the cross-section of U.S. stock returns. Hou, Karolyi, and Kho (2011) show the importance of the closely-related cash-flow-to-price characteristic in pricing international stocks.

and EMs. However, investment in EMs has lower average returns (1.72% compared to 4.02%). Notwithstanding the difference in sample period and cross-section of countries, the magnitude of our historical average returns for size, value, momentum, profitability, and investment is comparable to past studies. For example, the stronger investment effect in DMs compared to EMs is also found in Titman, Wei, and Xie (2013). They report an average annual investment return of 2% and 4% for, respectively, EMs and DMs.

We also graphically report factor average returns and volatilities from Panel C in Figure 1. All regional factor average returns are positive, except for size in Developed Europe and profitability in Latin America. Across regions, the average market excess return is the highest in North America and lowest in Asia Pacific. Asia Pacific has the highest value and lowest momentum historical average returns. Thus, momentum returns in Asia remain weaker than those around the world, as previously found in Griffin, Ji, and Martin (2003). The largest average return for size is in North America and the smallest in Developed Europe. North America has the highest average return for investment and the second highest after Emerging Asia for profitability. The correlation between regional factor average returns and volatilities is positive at 0.31 (unreported).

In the next section, we explore which combinations of these factors can explain the factor structure in individual stock returns.

# 5 Empirical results

This section contains our main empirical results. We start by investigating which models capture the factor structure in stock returns by estimating different models for each country. Then, we explore the significance of the risk premia of factors identified in the first step and of the pricing errors generated by different models. Finally, we analyze the determinants of time-varying factor exposures.

### 5.1 Which factors capture comovements in international stock returns?

Underlying the validity of our international APT and the consistency of the risk premia estimator is the assumption that residuals are weakly cross-sectionally correlated. Residuals could be too highly correlated if there is a missing equity factor, a missing currency factor as discussed in Section 2.1, or a misspecified dynamics for the risk factor loadings.[25] To empirically evaluate whether the weak cross-correlation assumption is verified in the data, we use the GOS2 diagnostic criterion, described in Appendix A.6, to determine whether there remains a factor structure in model residuals.

Figure 2 contains our results. We estimate several models, each with a different combination of factors, on each country and compute the proportion of negative diagnostic criteria across countries. A negative diagnostic criterion value indicates that the candidate set of factors has successfully captured all the factor structure in USD-denominated returns. A positive value otherwise indicates that we are missing at least one factor that drives stock returns, whether it be an omitted equity or currency factor. A higher proportion of negative values, as reported using bars in Figure 2 shows the success of each candidate model in capturing all important factors in the data.

The top (bottom) graphs report on proportions using models with world (regional) factors. The left column reports on proportions across DMs and the right column reports on EM proportions. As we move from the leftmost model (only a market factor) to the rightmost model ($q$-factor) in each graph, we sequentially add one factor to investigate their respective importance.

For each candidate set of factors, we find that world factors capture the factor structure in about 60% of DMs and 20% of EMs.[26] Regional factors fare better, capturing the factor structure in about 65-90% of DMs and 30% of EMs. Hence, neither world nor regional factors, on their own, are sufficient to capture the factors structure in most countries.

The second bar for each candidate set of factors in Figure 2 adds an excess country market factor to other factors. We construct the excess country market factor as the return of the country

[25] As discussed in GOS2, even if the omitted factors are not priced, the risk premia estimates of the priced factors will not converge to their true value, and biases on betas and risk premia will not compensate each other.

[26] Those inferred proportions are not subject to a multiple testing problem and do not require a Bonferroni correction or a false discovery approach (see, for example Barras, Scaillet, and Wermers, 2010; Bajgrowicz and Scaillet, 2012).

market factor in excess of the world market factor, $f_m^{Country} - f_m^{World}$, for models that use world factors. For models with regional factors, we compute the return in excess of the regional market factor, $f_m^{Country} - f_m^{Region}$.[27] We denote this factor as the *excess* country market factor and these models as mixed world and mixed regional models.

Across *all* sets of factors, adding the excess country market factor leads to proportions higher than 87% for DMs and 83% for EMs. Regional factors lead to slightly better performance. The regional four-factor model with market, size, value, and momentum, and the regional $q$-factor model with market, size, profitability, and investment, both augmented with the excess country market factor, explain the factor structure for all DMs.

The excess country market factor is the return on the local stock market denominated in the local currency converted to USD. Therefore, the importance of the excess country market factor could stem from the currency return. To address this question, we estimate all models by augmenting the candidate sets of world or regional factors with the country currency returns instead of the excess country market factor. The proportions for these models are reported with the next set of bars in Figure 2. While models with currency factor perform better than models without, they underperform models with the excess country market factor, especially in EMs. Therefore, there is a source of risk in the excess country market factor beyond the local currency required to capture the factor structure. Our results do not point to the irrelevance of currency risk factors. Rather, our results indicate that the excess country market factors capture currency risk as well as other sources of risk needed to explain stock return comovements.

To further explore the role of currency risk, we next consider systematic currency factors instead of using each country-specific currency returns. We add the currency carry factor of Lustig, Roussanov, and Verdelhan (2011) and the dollar factor of Verdelhan (2018) to each set of candidate factors. The last set of bars in Figure 2 indicates that these models perform almost as good as the

[27] Stehle (1977), Bekaert, Hodrick, and Zhang (2009) and Karolyi and Wu (2018) also propose mixed models with global and local factors, where the local factor is orthogonalized on the global factor. We use a simple return difference to avoid any look-ahead bias in the construction of a given factor, possibly induced by the projection coefficient estimated from the full sample. This construction choice also eases the interpretation of the total country market risk premia as the sum of the world (regional) market and the excess country market risk premia. Country factors in excess of the world and regional market factors have low cross-correlations, which support the fact that excess country market factors are key in capturing the factor structure in many countries.

ones with each country-specific currency returns. This result is not surprising; Verdelhan (2018) shows that these two factors explain a large proportion of the variations in currency returns. Brusa, Ramadorai, and Verdelhan (2015) show that a model with the global market factor and these two systematic currency factors outperforms other global multi-factor models. Their model would correspond to the fourth bar for "MKT" in the top row, with the exception that our world market factor is measured in USD and theirs is measured in local currency. We find that the proportion of DMs (EMs) whose factor structure is captured by this model is about 20% (50%) lower than the proportion obtained using only the world and excess country market factors (see the second bar in the same bar group).

To provide another perspective, we graphically report the correlation matrices of individual stock returns and residuals from the mixed world four-factor model. We compute the correlation matrix using all individual stocks that are kept in the model estimations. Of these stocks, we keep all pairs for which we have more than two years of overlapping monthly returns. In Figure 3, we report the average block correlations between countries. The top panel reports the cross-sectional average of the absolute value of the correlations of all stocks within a country. The blocks above the diagonal in the bottom panel are the cross-sectional average of the absolute value of the correlation between each stock in a country and each stock in another country. We order countries by region on both axes.

The highest country averages of the absolute value of the correlations are close to 0.45 and are found in Peru, Saudi Arabia, and China. Cross-country average of the absolute value of the correlations are rarely higher than 0.25. Most notably, we can observe in the upper part of the bottom panel the European block in which all countries exhibit high correlations between each other.

Return correlations sharply contrast with correlations of their mixed world four-factor model residuals reported on the second line of the top panel and below the diagonal in the bottom panel. Once we control for world market, size, value, momentum, and excess country market factors, all of the within- and cross-country averages of the absolute values of the correlations are close to 0. These low average absolute correlations provide a graphical perspective of the fact that we capture

the whole factor structure, including currency risk, present in USD-denominated returns. These small absolute correlations also indicate that the risk exposure dynamics are correctly specified in that they achieve weak cross-sectional dependence in the residuals.[28]

Results in this section speak to the importance of the excess country market factor in explaining comovements across individual stock returns. However, they do not imply that this factor is priced. Next, we examine the economic magnitude of the excess country market factor risk premia and how they compare to other global and regional factor risk premia. For the sake of space, we focus our discussion on the mixed world four-factor and $q$-factor models. We repeat all of our analyses in the Internet Appendix for the mixed regional four-factor and $q$-factor models.

### 5.2 Are factors priced in individual international stock returns?

We showed in the last section that multiple risk factors, including the excess country market factor, are essential to describe the comovements between stock returns. We now examine whether they carry a significant risk premium.

We report results in Table 2 on the significance of factor risk premia aggregated across DMs and EMs. For each factor, we report in columns *(i)* and *(ii)* the average risk premia. Because we standardize the common instruments, except the constant, to have mean zero in our empirical application, the value and $t$-ratio for the parameter for the constant, $\Lambda_0$, indicate the average risk premium and its significance (see Equation (6)). For the mixed world four-factor and $q$-factor models, we find positive risk premia for all but one factor, ranging from 0.04% per month for profitability in EMs to 1.44% for the excess country market in EMs. For the excess country market, we find larger risk premia for EMs than for DMs. The premium is -0.08% (0.29%) for the four-factor ($q$-factor) model for DMs and 1.44% (0.59%) for EMs.

In columns *(iii)* and *(iv)*, we report the proportion of DMs and EMs for which the risk premia are significant at the 5% level, using the $t$-ratio for the parameter $\Lambda_0$. We find that factor risk premia are significant in most countries, with proportions ranging from 42% for momentum in EMs to 83% for the excess country market and size in DMs.

[28] In the special case of the market model, the value weighted average covariance of residuals from a regression of returns on the value-weighted market factor is, by definition, equal to zero.

Using an instrumental variable estimator to estimate factor models on individual stocks in the U.S. market, Jegadeesh et al. (2019) find that almost all factor risk premia are insignificant whereas the corresponding stock characteristics are highly significant. Our results differ from theirs in one key aspect. Results in Table 2 show that the factor risk premia are significant in a large proportion of countries. Therefore, our results point to the importance of factor risk premia in individual stock returns, even controlling for stock characteristics.

Our diagnostics for the factor structure in the previous section indicate that the local market is key to explain the comovements of individual international stock returns. Further, our estimations also show that this factor carries a significant risk premium in a large proportion of countries and that its risk premium is much larger in EMs than in DMs. We report in Internet Appendix D results for the mixed regional models. All results are similar, except that average risk premia for the excess country market factors are smaller. However, they remain significant in a large proportion of countries.

Could we obtain the same conclusion about the importance of the excess country market factor from an unconditional factor spanning test? For each country, we run a regression of the excess country market factor on a constant and the other factors in the four-factor or $q$-factor model (see Barillas and Shanken, 2017; Fama and French, 2018, for similar factor spanning tests). In unreported results, we find that the intercept is insignificant for almost all countries and models. Although the other factors unconditionally span the excess country market factor, the latter has a significant role in capturing the time-varying factor structure of individual stocks as shown in Section 5.1 and in describing their expected returns. Therefore, empirical findings differ when using individual stock returns in conditional factor models instead of test portfolios (i.e., the excess country market) in unconditional factor models.

As discussed in Section 2.2, if the country market in excess of the world market earns a significant risk premium, then the country is segmented from the world market. Likewise, if we reject the null that the country market in excess of the regional market earns a zero risk premium, then the country is segmented from the region. Hence our results inferred from individual stocks provide

evidence of world and regional segmentation both in DMs and EMs.[29]

Past studies based on market indices and international equilibrium models show that country market risk is not priced in DMs (see, for example, De Santis and Gerard, 1997; Carrieri, Chaieb, and Errunza, 2013) but is priced in EMs (see, for example, Errunza and Losq, 1985; Bekaert and Harvey, 1995; Carrieri, Errunza, and Hogan, 2007; Carrieri, Chaieb, and Errunza, 2013). Our new results show that country market risk is priced for many DMs as for EMs.[30]

### 5.3 How important are pricing errors in international stock returns?

In this section, we examine the ability of factor risk premia to explain expected international stock returns. First, we proceed to formal asset pricing tests. For each country, we compute the test statistic and its $p$-value for the test of the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$, and for the case of tradable factors, $a_{c,t}(\gamma) = 0$. The latter corresponds to the traditional test that alphas from time-series regressions are jointly equal to zero (see Gibbons, Ross, and Shanken, 1989), but here with an inference suitable for large unbalanced panels.

In results reported in Internet Appendix E, we find small proportions of cases in which the mixed models are not rejected, meaning cases when the set of factors is correctly specified and the no-arbitrage asset pricing restrictions are not rejected.

Since rejecting the asset pricing restrictions indicate only that the pricing errors are too large in aggregate, we investigate the time-series dynamics of the pricing errors. Models could be rejected because pricing errors are too large on average or because they are small on average but dramatically spike during some specific periods.

[29] If the asset pricing restrictions do not hold, we can still test for the significance of a factor risk premium. GOS provide valid risk premia inferential analysis when asset pricing restrictions are rejected. See also our discussion in Section 5.3.

[30] Various authors have examined the issue of international capital market integration and suggested different measures. Bekaert and Harvey (1995) use regime-switching statistical models to examine the dynamics of world market integration. Carrieri, Errunza, and Hogan (2007) and Carrieri, Chaieb, and Errunza (2013) estimate a conditional version of the asset pricing model of Errunza and Losq (1985) and obtain a time-varying measure of integration. Eun and Lee (2010) document a mean-variance convergence in the risk-return distance among 17 international stock markets. Bekaert, Harvey, Lundblad, and Siegel (2011) suggest a new measure based on earnings multiples and show that DMs have been integrated since 1993 while EMs remain segmented. Pukthuanthong and Roll (2009) use the R-squared of linear factor models as a measure of integration. Eiling and Gerard (2015) measure integration as the fraction of total risk due to global factors. Errunza and Miller (2000) examine changes in equity valuations at the firm level following the introduction of depositary receipts and show a significant reduction in the cost of capital.

We decompose the expected return of a stock for month $t$ as,

$$E\left[r_{c,t}(\gamma)|\mathcal{F}_{t-1}\right] = \underbrace{a_{c,t}(\gamma) - b_{c,t}(\gamma)'\nu_{c,t}}_{\text{pricing error}} + \overbrace{b_{c,t}(\gamma)'\lambda_{c,t}}^{\text{factor risk premia}}. \tag{10}$$

Then, we compute the expected return of an equal-weighted portfolio by averaging the expected returns across all available stocks. If the asset pricing restrictions in Equation (4) are not rejected, the pricing error is equal to zero and the expected return comes only from the factor risk premia. In contrast, if asset pricing restrictions are rejected, the decomposition in Equation (10) shows that pricing errors, driven by common instruments and stock characteristics, contribute to expected returns beyond their relations to factor exposures. Since the asymptotics in GOS accommodate the possibility of zero and non-zero pricing errors, the empirical counterpart of the mispricing decomposition in Equation (10) converges to their population values regardless of whether asset pricing restrictions hold or not.[31]

We report in Figure 4 the time series of the cross-sectional average of pricing errors in yellow, factor risk premia in blue, and total expected return using a red line. We consider an equal-weighted portfolio of all available stocks for each of the three DM regions and EM regions. To better interpret the variations in the total expected return from our model estimates, we add the annualized average realized excess return over the next 24 months using a dashed black line. Figure 4 uses the mixed world four-factor model and Figure 5 uses estimates from the mixed world $q$-factor model.

Gray bars indicate recession periods. We use recession dates from the NBER for the U.S. and from the Economic Cycle Research Institute for non-U.S. countries. We build a recession indicator for each region, which is equal to one when at least half of the countries in the region are in a recession. We report in each figure gray areas to denote the recession periods using all regional recession indicators in the region (North America, Developed Europe, and Asia Pacific for DMs and Latin America, Emerging Europe, Middle East and Africa, and Emerging Asia for EMs).

[31] GOS shows that, even if the asset pricing restrictions do not hold, the second pass OLS estimator of $\nu_c$ has a well-defined limit. That limit is a pseudo-true value (White, 1982; Gourieroux et al., 1984) instead of a true value. The estimator of $\nu_c$ has a slower rate of convergence and a different asymptotic covariance matrix (see Propositions 5 and 7 in GOS).

Figure 4 shows important time-variations in aggregate pricing errors and factor risk premia. In particular, pricing errors are an important part of total expected returns. The time-series median of the absolute pricing error, standardized by the total expected return, ranges from 29% for Developed Europe to 63% for Developed Asia Pacific. The differences between the total expected return (red line) and the total factor risk premia (blue area) indicate the extent to which relying only on factor risk premia to obtain expected stock returns produces biased estimates of expected returns.

Figure 5 shows the equal-weighted portfolios' expected return and realized returns for the mixed world $q$-factor model. The results are similar. Both aggregate pricing errors and factor risk premia exhibit important variations over time. The time-series median of the absolute pricing error standardized by the total expected return ranges from 23% for Latin America to 41% for Emerging Europe, Middle-East, and Africa. Compared to the four-factor mixed model, the $q$-factor mixed model shows smaller pricing errors on average across DMs and EMs. Again, we see from the difference between the red line and the blue area that relying only on factor risk premia to get an estimate of expected returns is inaccurate. We also find similar results for mixed regional models (see Internet Appendix D).

### 5.4 Which instruments are important for time-varying factor exposures?

In our final analysis, we use our coefficient estimates to examine which variables drive the time-variations in factor exposures. We report in Table 3 the median coefficient value across stocks for each factor and each instrument in the mixed world four-factor model (when the regressor is selected). Below each median value, we report the proportion of stocks for which the regressor is selected. For each coefficient, we report the median and proportion across all DMs and across all EMs. Table 4 presents the results for the mixed world $q$-factor model and has the same structure as Table 3.

Four key points emerge from Tables 3-4. First, while the constant unsurprisingly has a large positive value close to one for the market and the excess country market factor, it also has a high positive value for the size factor. The intuition for this result is that small stocks equally weigh in

the computation of the median.

Second, the median coefficient values are the largest for the characteristics on which each factor is based; size and investment are negative for the size and investment factors, respectively, and value, momentum, and profitability are positive for the value, momentum, and profitability factors, respectively. The magnitude of these relations is smaller in EMs, especially for momentum and investment.

Third, except for the constant, which is always selected, the proportions of stocks for which regressors are selected range from 27% to 79%. While there are no regressors that are never selected, the proportions far below 100% demonstrate the need to optimally select instruments and extend the GOS methodology to an international setup.

Finally, we obtain median $R^2$s of around 0.20 for DMs and 0.35 for EMs, similar to the values obtained for the U.S. market in Gagliardini et al. (2019b). We obtain the same key results with mixed regional models as reported in Internet Appendix F.

# 6 Conclusion

We study the factor structure and estimate time-varying risk premia in international individual stock returns. Our international database includes $64,392$ stocks from 47 countries, presenting the largest cross-sectional dispersion in average returns that any asset pricing model for the stock market should seek to explain.

Based on a diagnostic criterion for approximate factor structures, we find that the excess country market factor is required in addition to world or regional market and non-market factors to capture the factor structure in stock returns for both DMs and EMs. The risk premia for these excess country market factors are significant in many countries and economically larger in EMs than in DMs. Our results also indicate that market and non-market factor risk premia are significantly positive in a large proportion of countries. Finally, we show that pricing errors are an economically large and time-varying part of total expected returns.

Our results show that country allocations continue to be an important consideration for man-

agers of global stock portfolios. There are other interesting applications of our methodology for fund managers. For example, we can decompose the contributions of each factor to total expected excess returns for global stock portfolios.

We can similarly decompose the contributions to expected returns coming from pricing errors. Our extension of the GOS methodology allows using a broad set of instruments that are not selected a priori, but by a data-driven sequential approach. A natural question for future research would be to investigate the drivers of pricing errors, beyond the instruments used in this paper, such as measures of liquidity, short-selling constraints, and other microstructure features. We leave this analysis for future work.

# Appendix A Estimation and test methodology

We detail in this section the estimation and test methodology with common instruments $Z_{c,t}$ for stocks of country $c$ and stock-specific instruments $Z_{c,t}(\gamma_{i,c})$ where $\gamma_{i,c}$ corresponds to a draw of asset $i$ in country $c$, $i = 1, ..., n_c$. For simplicity, we use in this Appendix the subscripts $i$ and $c$ to denote a stock in country $c$ instead of the index $\gamma_{i,c}$. Such a sampling scheme ensures that cross-sectional limits exist and are invariant to reordering of the assets (ses GOS for a discussion). Since observable assets are random draws from an underlying population (Andrews, 2005), we get a standard random coefficient model (see, for example, Hsiao, 2003, Chapter 6).

A major impediment to testing the IAPT with multiple factors and time-varying factor exposures and risk premia is the proliferation of parameters to estimate. To address this problem, we extend the methodology of Gagliardini, Ossola, and Scaillet (2016) in the following way. For each stock, we use an automatic procedure to select the most important instruments while making sure that model specification is consistent with no-arbitrage conditions for *each* stock. Then, we adjust the asset pricing test to account for the different number of parameters across stocks. We detail each step below.

## A.1 Time-series regressions

Our specification choices for factor exposures and factor risk premia (Equations (5), (6), and (7) in the main text) combined with the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)\nu_{c,t}$, imply that a stock intercept is $a_{i,c,t} = Z'_{c,t-1}B'_{i,c}\left(\Lambda_c - F_c\right)Z_{c,t-1} + Z'_{i,c,t-1}C'_{i,c}\left(\Lambda_c - F_c\right)Z_{c,t-1}$.

To avoid the curse of dimensionality, we impose some structure and set some elements in $B_{i,c}$ and $C_{i,c}$ to zero. Let $\mathbb{I}_{B_{i,c}}$ and $\mathbb{I}_{C_{i,c}}$ be a $K$-by-$p$ and a $K$-by-$q$ indicator matrices whose elements are equal to one if the corresponding elements in $B_{i,c}$ and $C_{i,c}$ are not zero. The $i$ subscript indicates that the indicator matrices can change across stocks and we explain in Section A.2 how they are selected.

We further define $\tilde{\mathbb{I}}_{B_{i,c}}$ as the $p$-vector whose $j^{th}$ element is equal to one if at least one element in the $j^{th}$ column of $B_{i,c}$ is not zero and equal to zero otherwise. Let $\tilde{p}_i = \tilde{\mathbb{I}}'_{B_{i,c}}\iota_p$ be the number of

columns in $B_{i,c}$ with at least one non-zero element where $\iota_p$ be a $p$-vector of ones.

We extend the GOS methodology in our high-dimensional international setting using these indicator matrices to reduce the number of parameters to estimate. One possibility would be to compute the Hadamard products, $\mathbb{I}_{B_{i,c}} \circ B_{i,c}$ and $\mathbb{I}_{C_{i,c}} \circ C_{i,c}$ and use them in the time-series regressions. However, this approach, based on simple element-by-element products, would create regressors whose values are all zeros and would prevent running the time-series regressions. Instead, to get feasible least-squares estimations, we use these indicator matrices to select the required regressors in a parsimonious way as follows.

We impose some structure on the use of common and stock-specific instruments. First, all elements in the first column of $\mathbb{I}_{B_{i,c}}$ are equal to one, i.e., we always use a constant to model factor exposures and $\tilde{p}_i \geq 1$. Those exposures correspond to the time-invariant factor contributions. Second, at least one element in each column of $\mathbb{I}_{C_{i,c}}$ is equal to one to ensure all stock-specific instruments are relevant for at least one factor. We do not impose *a priori* any structure on the matrix $\Lambda_c - F_c$. Our approach is general as it only requires specifying the elements in $B_{i,c}$ and $C_{i,c}$ as long as the above two restrictions are respected.

To simplify the notation, we define the $d_{1,i} = p(p-1)/2 + \tilde{p}_i + pq$ vector of predetermined variables

$$x_{i,c,t,1} = \left(\text{vech}(X_t)'\tilde{D}_{i,c}, Z'_{c,t-1} \otimes Z'_{i,c,t-1}\right)', \tag{A.1}$$

where the matrix $X_t$ has typical diagonal elements $X_{k,k,t} = Z^2_{c,t-1,k}$ and off-diagonal elements $X_{k,l,t} = 2Z_{c,t-1,k}Z_{c,t-1,l}$, $\tilde{D}_{i,c}$ is the matrix $diag\left(\text{vech}\left[diag\left(\tilde{\mathbb{I}}_{B_{i,c}}\right) + \iota_p\iota'_p - I_p\right]\right)$ in which columns with all 0s have been removed, and where $I_p$ is the identity matrix of size $p$. Even if a common instrument is never selected to model factor exposures, it will still appear in the term $\text{vech}(X_t)'\tilde{D}_{i,c}$ because of its presence in the matrix $\Lambda_c - F_c$.

In addition, we define the $d_{2,i} = (n_{\mathbb{I}_{B_{i,c}}} + n_{\mathbb{I}_{C_{i,c}}})$-vector of factors scaled by $Z_{c,t-1}$ (scaled factors) and by $Z_{i,c,t-1}$

$$x_{i,c,t,2} = \left[\left(f'_t \otimes Z'_{c,t-1}\right)\tilde{B}'_{i,c}, \left(f'_t \otimes Z'_{i,c,t-1}\right)\tilde{C}'_{i,c}\right]', \tag{A.2}$$

where $n_{\mathbb{I}_{B_{i,c}}}$ and $n_{\mathbb{I}_{C_{i,c}}}$ are respectively the number of non-zero elements in $B_{i,c}$ and $C_{i,c}$. In this

equation, the $n_{\mathbb{I}_{B_{i,c}}}$-by-$Kp$ matrix $\tilde{B}_{i,c}$ is obtained by removing the rows in $diag\left(vec\left[\mathbb{I}'_{B_{i,c}}\right]\right)$ for which all elements are equal to zero. Similarly, the $n_{\mathbb{I}_{C_{i,c}}}$-by-$Kq$ matrix $\tilde{C}_{i,c}$ is obtained by removing the rows in $diag\left(vec\left[\mathbb{I}'_{C_{i,c}}\right]\right)$ for which all elements are equal to zero. The new definitions of $x_{i,c,t,1}$ and $x_{i,c,t,2}$ relying on the new matrices $\tilde{B}_{i,c}$ and $\tilde{C}_{i,c}$ clarify the differences with the GOS setting.

Then, we can use the compact notation with the $d_i$-vector $x_{i,c,t} = \left(x'_{i,c,t,1}, x'_{i,c,t,2}\right)'$ with $d_i = d_{1,i} + d_{2,i}$,

$$r_{i,c,t} = \beta'_{i,c} x_{i,c,t} + \varepsilon_{i,c,t}, \tag{A.3}$$

with $\beta_{i,c} = \left(\beta'_{i,c,1}, \beta'_{i,c,2}\right)'$, and

$$\begin{aligned}
\beta_{i,c,1} &= \left(\left(X' N_p \left[(\Lambda_c - F_c)' \otimes I_p\right] \tilde{B}'_{i,c} \tilde{B}_{i,c} vec\left[B'_{i,c}\right]\right)', \left(\left[(\Lambda_c - F_c)' \otimes I_q\right] vec\left[C'_{i,c}\right]\right)'\right)', \\
\beta_{i,c,2} &= \left(\left(\tilde{B}_{i,c} vec\left[B'_{i,c}\right]\right)', \left(\tilde{C}_{i,c} vec\left[C'_{i,c}\right]\right)'\right)',
\end{aligned}$$

where $N_p = \frac{1}{2} D_p^+ \left(W_{p,p} + I_{p^2}\right)$, $W_{p,q}$ is the commutation matrix such that $\text{vec}(A') = W_{p,q}\text{vec}(A)$ for a $p$-by-$q$ matrix $A$, and $D_p^+$ is the $p(p+1)/2$-by-$p^2$ matrix such that $\text{vech}(A) = D_p^+ \text{vec}(A)$. Here the dimension $d_i$ of the vector $x_{i,c,t}$ depends on asset $i$ while it does not in GOS. The dimension of their parameter vector $\beta_{i,c}$ is held fixed across stocks $d_i = d$. The new representation (A.3) induced by our international setting ensures that the time-series regression specification used in the first pass is compatible with no-arbitrage conditions. Removing some regressors in $x_{i,c,t,2}$ without removing the associated regressors in $x_{i,c,t,1}$ could introduce arbitrage.

We account for the unbalanced nature of the panel through a collection of indicator variables: we define $I_{i,c,t} = 1$ if the return of asset $i$ in country $c$ is observable at date $t$, and 0 otherwise (Connor and Korajczyk, 1987). The first pass consists in computing time-series OLS estimators $\hat{\beta}_{i,c} = \hat{Q}^{-1}_{x,i,c} \frac{1}{T_{i,c}} \sum_t I_{i,c,t} x_{i,c,t} r_{i,c,t}$, for all stocks $i = 1, ..., n_c$, where $\hat{Q}_{x,i,c} = \dfrac{1}{T_{i,c}} \displaystyle\sum_t I_{i,c,t} x_{i,c,t} x'_{i,c,t}$ and $T_{i,c} = \displaystyle\sum_t I_{i,c,t}$.

## A.2 Stock and instrument selection

The random sample size $T_{i,c}$ for stock $i$ in country $c$ can be small, and the inversion of matrix $\hat{Q}_{x,i,c}$ can be numerically unstable, possibly yielding unreliable estimates of $\beta_{i,c}$. Also, given that we use the cross-sectional ranks of characteristics as stock-specific instruments, we encounter many cases of multicollinearity. Consider for example a stock that remains among the largest stocks in its market during the sample period. Then its size cross-sectional rank, $Z^{size}_{i,c,t-1}$, is relatively constant and the interaction terms $Z^{size}_{i,c,t-1} f_{c,t}$ are highly correlated with $f_{c,t}$ for all factors in the regression.

To address this problem, we first keep only stocks with at least five years of monthly returns, $T_{i,c} \geq 60$. Then, we select instruments for the factor exposures to obtain a time-series regression that is well-conditioned enough. We follow these steps:

1. As a criterion, we use the condition number which is the square root of the ratio of the largest eigenvalue to the smallest eigenvalue of $\hat{Q}_{x,i,c}$, $CN\left(\hat{Q}_{x,i,c}\right) = \sqrt{eig_{\max}\left(\hat{Q}_{x,i,c}\right) / eig_{\min}\left(\hat{Q}_{x,i,c}\right)}$. A too large value of $CN\left(\hat{Q}_{x,i,c}\right)$ indicates multicollinearity problems and ill-conditioning (Belsley et al., 2004; Greene, 2008). In our empirical tests, we use a threshold of 15.

2. If the condition number for stock $i$ is above 15, then we find the pair of regressors in $x_{i,c,t,2}$ that has the largest cross-correlation in absolute value. Of these two regressors, we remove the one with the lowest absolute correlation with $r_{i,c,t}$ and set its corresponding element in $\mathbb{I}_{B_{i,c}}$ or $\mathbb{I}_{C_{i,c}}$ to 0.

3. We check that the two conditions on the specification are respected, namely that the first column of $B_{i,c}$ are all selected and that there is at least one element selected in each of the columns of $C_{i,c}$. Otherwise, we keep both regressors and look for the next regressor pair with the highest absolute cross-correlation.

4. Given our new selection of regressors $x_{i,c,t,2}$, we compute the new matrix $\tilde{D}_{i,c}$ and use Equation (A.1) to obtain the new selected $x_{i,c,t,1}$. Therefore, our methodology selects instruments for the factors exposures and ensures that the time-series regression specification is consistent with no-arbitrage conditions.

We follow these steps until the condition number falls below 15. If no specification respects the two specification conditions and has a low enough condition number, then we do not keep stock $i$. We then define the indicator variable $\mathbf{1}^{\chi}_{i,c}$, which takes a value of one if stock $i$ is kept and zero otherwise.

In our empirical tests, we have found that this sequential procedure produces better results in terms of time-series fit and pricing errors than selecting *a priori* the instruments to use for each factor.

### A.3 Cross-sectional regressions

The second pass consists of computing a cross-sectional estimator of $\nu_c$ by regressing the $\hat{\beta}_{i,c,1}$s on the $\hat{\beta}_{i,c,3}$s keeping only the non-trimmed assets. First, we can write $\beta_{i,c,1}$ as

$$\beta_{i,c,1} = \beta_{i,c,3}\nu_c, \tag{A.4}$$

where

$$\beta_{i,c,3} = \left(\left(\tilde{D}'_{i,c}N_p\left[B'_{i,c}\otimes I_p\right]\right)', \left[W_{p,q}\left(C'_{i,c}\otimes I_p\right)\right]'\right)', \tag{A.5}$$

and

$$\nu_c = vec\left[\Lambda'_c - F'_c\right]. \tag{A.6}$$

We obtain $\beta_{i,c,3}$ using the following identity coming from our extension of the GOS methodology,

$$\begin{aligned}
\mathrm{vec}(\beta'_{i,c,3}) &= J_{a,i}\beta_{i,c,2}, \\
J_{a,i} &= \begin{pmatrix} J_{1,i} & 0 \\ 0 & J_{2,i} \end{pmatrix}, \\
J_{1,i} &= W_{p(p-1)/2+\tilde{p}_i,Kp}\left(I_{pK}\otimes\left(\tilde{D}'_{i,c}N_p\right)\right)\left\{I_K\otimes\left[\left(W_p\otimes I_p\right)\left(I_p\otimes vec\left[I_p\right]\right)\right]\right\}\tilde{B}'_{i,c}, \\
J_{2,i} &= W_{pq,pK}\left(I_K\otimes\left[(I_p\otimes W_{p,q})(W_{p,q}\otimes I_p)(I_q\otimes \mathrm{vec}(I_p))\right]\right)\tilde{C}'_{i,c}.
\end{aligned}$$

We use a Weighted Least Squares (WLS) approach,

$$\hat{\nu}_c^{WLS} = \hat{Q}_{\beta_3}^{-1} \frac{1}{n_c} \sum_i \hat{\beta}'_{i,c,3} \hat{w}_{i,c} \hat{\beta}_{i,c,1}, \tag{A.7}$$

where $\hat{Q}_{\beta_3} = \frac{1}{n_c} \sum_i \hat{\beta}'_{i,c,3} \hat{w}_{i,c} \hat{\beta}_{i,c,3}$ and $\hat{w}_{i,c} = \mathbf{1}_{i,c}^{\chi} (diag[\hat{v}_{i,c}])^{-1}$ are the weights.

The terms $v_{i,c} = \tau_{i,c} C'_{\nu_c,i} Q_{x,i,c}^{-1} S_{ii,c} Q_{x,i,c}^{-1} C_{\nu_c,i}$ are the asymptotic variances of the standardized errors $\sqrt{T}\left(\hat{\beta}_{i,c,1} - \hat{\beta}_{i,c,3}\nu_c\right)$ in the cross-sectional regression for large $T$, where $\tau_{i,c} = E[I_{i,c,t}|\gamma_{i,c}]^{-1}$, $Q_{x,i,c} = E\left[x_{i,c,t} x'_{i,c,t}\right]$, $S_{ii,c} = E\left[\varepsilon_{i,c,t}^2 x_{i,c,t} x'_{i,c,t} | \gamma_{i,c}\right]$, $C_{\nu_c,i} = (E'_{1,i} - (I_{d_{1,i}} \otimes \nu'_c) J_{a,i} E'_{2,i})'$, $E_{1,i} = \left(I_{d_{1,i}}, \mathbf{0}_{d_{1,i},d_{2,i}}\right)'$, $E_{2,i} = \left(\mathbf{0}_{d_{2,i},d_{1,i}}, I_{d_{2,i}}\right)'$, and $\mathbf{0}_{d_{1,i},d_{2,i}}$ is a $d_{1,i}$-by-$d_{2,i}$ matrix of zeros.

To operationalize this WLS approach, we first estimate $\hat{\nu}_c^{OLS}$ by OLS using unit weights $\hat{w}_{i,c} = 1$. We then use the estimates $\tau_{i,c} = \frac{T_c}{T_{i,c}}$, $C_{\hat{\nu}_c,i} = (E'_{1,i} - (I_{d_{1,i}} \otimes \hat{\nu}_c^{OLS'}) J_{a,i} E'_{2,i})'$, $\hat{S}_{ii,c} = \frac{1}{T_{i,c}} \sum_t I_{i,c,t} \hat{\varepsilon}_{i,c,t}^2 x_{i,c,t} x'_{i,c,t}$, and $\hat{\varepsilon}_{i,c,t} = r_{i,c,t} - \hat{\beta}'_{i,c} x_{i,c,t}$ to estimate $\hat{\nu}_c^{WLS}$ by WLS.[32]

The distribution of $\hat{\nu}_c^{WLS}$ is

$$\sqrt{n_c T_c}\left(\hat{\nu}_c^{WLS} - \frac{1}{T_c}\hat{B}_{\nu_c} - \nu_c\right) \Rightarrow N\left(0, \Sigma_{\nu_c}\right), \tag{A.8}$$

where the presence of the bias term $\hat{B}_{\nu_c}$ comes from the well-known Error-In-Variable problem, that is, factor exposures are estimated with errors in the first step time-series regressions. We report the expressions for the bias term $\hat{B}_{\nu_c}$ and the estimation methodology for the covariance matrix $\Sigma_{\nu_c}$ in the following two sections.

To obtain estimates of the time-varying risk premia, $\hat{\lambda}_{c,t}$, we first obtain estimates of $F_c$ by a SUR regression of factors $f_{c,t}$ on lagged common instruments $Z_{c,t-1}$. Then, we obtain $\hat{\Lambda}_c$ through the relation $\nu_c = \text{vec}\left(\Lambda'_c - F'_c\right)$ and $\hat{\lambda}_{c,t} = \hat{\Lambda}_c Z_{c,t-1}$.

[32] In their additional empirical results, GOS show that a value-weighting scheme does not change point estimate values but can increase confidence intervals due to a precision loss.

## A.4 Estimation of the risk premium bias

The bias term for the estimate $\hat{\nu}_c^{WLS}$ of the risk premia is estimated as

$$\hat{B}_{\nu_c} = \hat{Q}_{\beta_3}^{-1} \frac{1}{n_c} \sum_{i=1} \tau_{i,c} J_{b,i} \text{vec}\left( E'_{2,i} \hat{Q}_{x,i,c}^{-1} \hat{S}_{ii,c} \hat{Q}_{x,i,c}^{-1} C_{\hat{\nu}_c} \hat{w}_{i,c} \right), \tag{A.9}$$

with $J_{b,i} = (\text{vec}(I_{d_{1,i}})' \otimes I_{Kp})(I_{d_{1,i}} \otimes J_{a,i})$.

## A.5 Estimation of the risk premium covariance matrix

The covariance matrix for the risk premium estimates $\hat{\nu}_c^{WLS}$ is estimated as $\hat{\Sigma}_{\nu_c} = \hat{Q}_{\beta_3}^{-1} \hat{S} \hat{Q}_{\beta_3}^{-1}$, where $\hat{S} = \frac{1}{n_c} \sum_{i,j} \frac{\tau_i \tau_j}{\tau_{i,j}} \beta'_{i,c,3} w_{i,c} C'_{\hat{\nu}_c,i} \hat{Q}_{x,i,c}^{-1} \tilde{S}_{ij,c} \hat{Q}_{x,j,c}^{-1} C_{\hat{\nu}_c,j} w'_{j,c} \beta_{j,c,3}$, $\tau_{i,j,c} = \frac{T_c}{T_{ij,c}}$, and $T_{ij,c} = \sum_t I_{i,c,t} I_{j,c,t}$.

In the above equation, we use a hard thresholded estimator, $\tilde{S}_{ij,c} = \hat{S}_{ij,c} \mathbf{1}_{||\hat{S}_{ij,c}|| \geq \kappa_{n_c,T_c}}$, where $\hat{S}_{ij,c} = \frac{1}{T_{ij,c}} \sum_t I_{i,c,t} I_{j,c,t} \hat{\varepsilon}_{i,c,t} \hat{\varepsilon}_{j,c,t} x_{i,c,t} x'_{j,c,t}$, $||\hat{S}_{ij,c}||$ is the Frobenius norm, $\kappa_{n_{c,T_c}} = M \sqrt{\frac{log(n_c)}{T_c}}$ is a data-dependent threshold, and $M$ is a positive number set by cross-validation (see GOS for details).

Thresholding ensures that the estimator $\hat{S}$ is consistent. Indeed, $\hat{S}$ involves a sum on $i$ and $j$ but is standardized only by $n_c$ (and not $n_c^2$). Consequently, the usual sample estimator is not consistent. We use in the definition of $\tilde{S}_{ij,c}$ the threshold proposed in Bickel and Levina (2008) and extended by GOS to a random coefficient setting,

## A.6 Diagnostic criterion for the factor structure

We compute the $T_c$-by-$T_c$ matrix $\Upsilon = \sum_{i=1}^{n_c} \mathbf{1}_{i,c}^{\chi} \bar{\varepsilon}_{i,c,t} \bar{\varepsilon}'_{i,c,t}$, where $T_c$ is the number of periods and $\bar{\varepsilon}_{i,c,t}$ is a $T_c$-by-one vector of standardized residuals $\bar{\varepsilon}_{i,c,t} = \frac{I_{i,c,t} \varepsilon_{i,c,t}}{\sqrt{\frac{1}{T_c} \sum_{t=1}^{T_c} I_{i,c,t} \varepsilon_{i,c,t}^2}}$.

The diagnostic criterion is given by

$$\zeta = eig_{\max}\left( \frac{\Upsilon}{n_c T_c} \right) - g\left(n_c, T_c\right), \tag{A.10}$$

where $g\left(n_c, T_c\right) = -P\kappa ln\left(\kappa\right)$ is a penalty term with $\kappa = \frac{\left(\sqrt{n_c} + \sqrt{T_c}\right)^2}{n_c T_c}$ and $P$ is a data-driven constant. We use a simulation-based method to select $P$ (see Appendix 7 in GOS2 for details and Monte Carlo results for unbalanced panels).

## A.7 Asset pricing test

The test for asset pricing restrictions is based on the weighted sum of squared residuals $\hat{Q}_e = \frac{1}{n_c}\sum_i \hat{e}'_{i,c}\hat{w}_{i,c}\hat{e}_{i,c}$, where $\hat{e}_{i,c} = \hat{\beta}_{i,c,1} - \hat{\beta}_{i,c,3}\hat{\nu}_c^{Unbiased}$. The distribution of the re-centered sum of squared residuals is

$$\tilde{\Sigma}_e^{-1/2} T_c \sqrt{n_c}\left(\hat{Q}_e - \frac{\bar{d}_1}{T_c}\right) \sim N(0,1)$$

where $\tilde{\Sigma}_e = \frac{2}{n_c}\sum_{i,j}\frac{\tau_{i,c}^2\tau_{j,c}^2}{\tau_{i,j,c}^2} Tr\left[\left(C'_{\hat{\nu}_c,i}\hat{Q}_{x,i}^{-1}\tilde{S}_{ij}\hat{Q}_{x,j}^{-1}C_{\hat{\nu}_c,j}\right)\hat{w}_{j,c}\left(C'_{\hat{\nu}_c,j}\hat{Q}_{x,j}^{-1}\tilde{S}_{ji}\hat{Q}_{x,i}^{-1}C_{\hat{\nu}_c,i}\right)\hat{w}_{i,c}\right]$ and $\bar{d}_1 = \frac{1}{n_c}\sum_{i=1}^{n_c} d_{1,i}$. This test is an extension of the one in GOS to accommodate our international setting with different dimensions $d_{1,i}$ across stocks due to our selection procedure of characteristics (see Section A.2).

## A.8 Distribution of the risk premium dynamic parameters $\Lambda_c$

The parameters for the dynamics of the risk premia, $\Lambda_c$, follow a normal distribution $\sqrt{T_c}vec[\hat{\Lambda}'_c - \Lambda'_c] \sim N(0, \Sigma_{\Lambda_c})$, where $\Sigma_{\Lambda_c} = \left(\mathbb{I}_K \otimes Q_z^{-1}\right)\Sigma_u\left(\mathbb{I}_K \otimes Q_z^{-1}\right), \Sigma_u = E\left[u_t u'_t \otimes Z_{c,t-1}Z'_{c,t-1}\right], u_t = f_{c,t} - F_c Z_{c,t-1}, Q_z = E\left[Z_{c,t-1}Z'_{c,t-1}\right]$.

## References


Adler, Michael, and Bernard Dumas, 1983, International portfolio choice and corporation finance: A synthesis, *Journal of Finance* 38, 925–984.

Ahn, Seung C., and Alex R. Horenstein, 2013, Eigenvalue ratio test for the number of factors, *Econometrica* 81, 1203–1227.

Andrews, Donald W. K., 2005, Cross-section regression with common shocks, *Econometrica* 73, 1551–1585.

Ang, Andrew, 2014, *Asset management: a systematic approach to factor investing* (Oxford University Press, New York, NY).

Ang, Andrew, Jun Liu, and Krista Schwarz, 2020, Using stocks or portfolios in tests of factor models, *Journal of Financial and Quantitative Analysis* 55, 709–750.

Asness, Clifford S., and Andrea Frazzini, 2013, The devil in HML's details, *Journal of Portfolio Management* 39, 49–68.

Asness, Clifford S., Andrea Frazzini, and Lasse Heje Pedersen, 2019, Quality minus junk, *Review of Accounting Studies* 24, 34–112.

Asness, Clifford S., Tobias J. Moskowitz, and Lasse Heje Pedersen, 2013, Value and momentum everywhere, *Journal of Finance* 68, 929–986.

Avramov, Doron, and Tarun Chordia, 2006, Asset pricing models and financial market anomalies, *The Review of Financial Studies* 19, 1000–1040.

Bai, Jushan, and Serena Ng, 2002, Determining the number of factors in approximate factor models, *Econometrica* 70, 191–221.

Bai, Jushan, and Serena Ng, 2006, Confidence intervals for diffusion index forecasts and inference for factor-augmented regressions, *Econometrica* 74, 1133–1150.

Bajgrowicz, Pierre, and Olivier Scaillet, 2012, Technical trading revisited: False discoveries, persis-

tence tests, and transaction costs, *Journal of Financial Economics* 106, 473–491.

Ball, Ray, Joseph Gerakos, Juhani T. Linnainmaa, and Valeri Nikolaev, 2016, Accruals, cash flows, and operating profitability in the cross section of stock returns, *Journal of Financial Economics* 121, 28–45.

Barillas, Francisco, and Jay Shanken, 2017, Which alpha?, *Review of Financial Studies* 30, 1316–1338.

Barillas, Francisco, and Jay Shanken, 2018, Comparing asset pricing models, *Journal of Finance* 73, 715–754.

Barras, Laurent, Olivier Scaillet, and Russ Wermers, 2010, False discoveries in mutual fund performance: measuring luck in estimated alphas, *Journal of Finance* 65, 179–216.

Bekaert, Geert, and Campbel R. Harvey, 1995, Time-varying world market integration, *Journal of Finance* 50, 403–444.

Bekaert, Geert, Campbel R. Harvey, Chris Lundblad, and Stephan Siegel, 2011, What segments equity markets?, *Review of Financial Studies* 42, 3841–3890.

Bekaert, Geert, Robert J. Hodrick, and Xiaoyan Zhang, 2009, International stock return comovements, *Journal of Finance* 64, 2591–2626.

Bekaert, Geert, and Michael S. Urias, 1996, Diversification, integration, and emerging market closed-end funds,, *Journal of Finance* 51, 835–870.

Belsley, David A., Edwin Kuh, and Roy E. Welsch, 2004, *Regression diagnostics - Identifying influential data and sources of collinearity* (John Wiley & Sons, New York).

Bickel, Peter J., and Elizaveta Levina, 2008, Covariance regularization by thresholding, *The Annals of Statistics* 36, 2577–2604.

Black, Fischer, Michael C. Jensen, and Myron Scholes, 1972, *The Capital Asset Pricing Model: Some empirical findings.* (In Jensen, M.C. (Ed.), Studies in the Theory of Capital Markets. Praeger, New York).

Brusa, Francesca, Tarun Ramadorai, and Adrien Verdelhan, 2015, The international capm redux, *Saïd Business School WP 2014-11* .

Carrieri, Francesca, Ines Chaieb, and Vihang Errunza, 2013, Do implicit barriers matter for globalization, *Review of Financial Studies* 26, 1694–1739.

Carrieri, Francesca, Vihang Errunza, and Kedreth Hogan, 2007, Characterizing world market integration through time, *Journal of Financial Quantitative Analysis* 42, 915–940.

Cattaneo, Matias D., Richard K. Crump, Max H. Farrell, and Ernst Schaumburg, 2017, Characteristic-sorted portfolios: Estimation and inference, *Forthcoming in the Review of Economics and Statistics* .

Chamberlain, Gary, and Michael Rothschild, 1983, Arbitrage, factor structure, and mean-variance analysis on large asset markets, *Econometrica* 51, 1281–1304.

Cho, D. Chinhyung, Cheol S. Eun, and Lemma W. Senbet, 1986, International arbitrage pricing theory: An empirical investigation, *Journal of Finance* 41, 313–329.

Christoffersen, Peter, Vihang Errunza, Kris Jacobs, and Hugues Langlois, 2012, Is the potential for international diversification disappearing? A dynamic copula approach, *Review of Financial Studies* 25, 3711–3751.

Cochrane, John H., 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy* 104, 572–621.

Cochrane, John H., 2005, *Asset pricing* (Princeton University Press, Princeton).

Connor, Gregory, Matthias Hagmann, and Oliver Linton, 2012, Efficient semiparametric estimation of the Fama-French model and extensions, *Econometrica* 80, 713–754.

Connor, Gregory, and Robert A. Korajczyk, 1987, Estimating pervasive economic factors with missing observations, *Working Paper No. 34, Department of Finance, Northwestern University* .

Cooper, Ilan, Andreea Mitrache, and Richard Priestley, 2017, A global macroeconomic risk model for value, momentum, and other asset classes, *Working paper, BI Norwegian Business School* .

Cooper, Ilan, and Richard Priestley, 2011, Real investment and risk dynamics, *Journal of Financial Economics* 101, 182–205.

De Moor, Lieven, and Piet Sercu, 2013, The smallest firm effect: An international study, *Journal of International Money and Finance* 32, 129–155.

De Roon, Frans A., Theo E. Nijman, and Bas J. M. Werker, 2001, Testing for mean-variance spanning with short sales constraints and transaction costs: The case of emerging markets, *Journal of Finance* 56, 721–742.

De Santis, Giorgio, and Bruno Gerard, 1997, International asset pricing and portfolio diversification with time-varying risk, *Journal of Finance* 52, 1881–1912.

De Santis, Giorgio, and Bruno Gerard, 1998, How big is the premium for currency risk?, *Journal of Financial Economics* 49, 375–412.

Dumas, Bernard, Karen K. Lewis, and Emilio Osambela, 2017, Differences of opinion and international equity markets, *Review of Financial Studies* 30, 750–800.

Dumas, Bernard, and Bruno Solnik, 1995, The world price of foreign exchange risk, *Journal of Finance* 50, 445–479.

Eiling, Esther, and Bruno Gerard, 2015, Emerging equity market comovements: Trends and macroeconomic fundamentals, *Review of Finance* 19, 15431585.

Errunza, Vihang, and Etienne Losq, 1985, International asset pricing under mild segmentation: Theory and test, *Journal of Finance* 40, 105–124.

Errunza, Vihang, and Darius P. Miller, 2000, Market segmentation and the cost of capital in international equity markets, *Journal of Financial and Quantitative Analysis* 35, 577–600.

Eun, Cheol S., Sandy Lai, Frans de Roon, and Zhe Zhang, 2010, International diversification with factor funds, *Management Science* 56, 1500–1518.

Eun, Cheol S., and Jinsoo Lee, 2010, Mean-variance convergence around the world, *Journal of Banking and Finance* 34, 856–870.

Fama, Eugene F., and Kenneth R. French, 1998, Value versus growth: The international evidence, *Journal of Finance* 53, 1975–1999.

Fama, Eugene F., and Kenneth R. French, 2012, Size, value, and momentum in international stock returns, *Journal of Financial Economics* 105, 457–472.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and Kenneth R. French, 2017, International tests of a five-factor asset pricing model, *Journal of Financial Economics* 123, 441–463.

Fama, Eugene F., and Kenneth R. French, 2018, Choosing factors, *Journal of Financial Economics* 128, 234–252.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 81, 607–36.

Feng, Guanhao, Stefano Giglio, and Dacheng Xiu, 2019, Taming the factor zoo, *Forthcoming in the Journal of Finance* .

Ferson, Wayne E., and Campbel R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.

Ferson, Wayne E., and Campbel R. Harvey, 1993, The risk and predictability of international equity returns, *Review of Financial Studies* 6, 527–566.

Freyberger, Joachim, Andreas Neuhierl, and Michael Weber, 2018, Dissecting characteristics nonparametrically, *Forthcoming in the Review of Financial Studies* .

Gagliardini, Patrick, Elisa Ossola, and Olivier Scaillet, 2016, Time-varying risk premium in large cross-sectional equity datasets, *Econometrica* 84, 985–1046.

Gagliardini, Patrick, Elisa Ossola, and Olivier Scaillet, 2019a, A diagnostic criterion for approximate factor structure, *Journal of Econometrics* 22, 503–521.

Gagliardini, Patrick, Elisa Ossola, and Olivier Scaillet, 2019b, *Forthcoming in the Handbook of*

*Econometrics*, volume 7.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Giglio, Stefano, and Dacheng Xiu, 2019, Asset pricing with omitted factors, *Working paper, University of Chicago* .

Gourieroux, Christian, Alain Monfort, and Alain Trognon, 1984, Pseudo maximum likelihood methods: theory, *Econometrica* 52, 681–700.

Greene, William H., 2008, *Econometric Analysis, 6th ed.* (Prentice Hall).

Griffin, John M., 2002, Are the Fama and French factors global or country specific?, *Review of Financial Studies* 15, 783–803.

Griffin, John M., Xiuqing Ji, and J. Spencer Martin, 2003, Momentum investing and business cycle risk: evidence form pole to pole, *Journal of Finance* 58, 2515–2547.

Hansen, Lars Peter, and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587–613.

Harvey, Campbell R., 1991, The world price of covariance risk, *Journal of Finance* 46, 111–157.

Heston, Steven L., K. Geert Rouwenhorst, and Roberto E. Wessels, 1995, The structure of international stock returns and the integration of capital markets, *Journal of Empirical Finance* 2, 173–197.

Hou, Kewei, G. Andrew Karolyi, and Bong-Chan Kho, 2011, What factors drive global stock returns?, *Review of Financial Studies* 24, 2527–2574.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: an investment approach, *Review of Financial Studies* 28, 650–705.

Hsiao, Cheng, 2003, *Analysis of Panel Data* (Econometric Society Monographs, 2nd edition, Cambridge University Press).

Ikeda, Shinsuke, 1991, Arbitrage asset pricing under exchange risk, *Journal of Finance* 46, 447–455.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3–53.

Jegadeesh, Narasimhan, Joonki Noh, Kuntara Pukthuanthong, Richard Roll, and Junbo Wang, 2019, Empirical tests of asset pricing models with individual assets: resolving the errors-in-variables bias in risk premium estimation, *Journal of Financial Economics* 133, 273–298.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Karolyi, G. Andrew, 2015, *Cracking the emerging markets enigma* (Oxford University Press, New York, NY).

Karolyi, G. Andrew, and René Stulz, 2003, *Are assets priced locally or globally?* (Handbook of the Economics of Finance, eds G. Constantinides, M. Harris and R. Stulz, Elsevier).

Karolyi, G. Andrew, and Ying Wu, 2017, Another look at currency risk in international stock returns, *Working paper, Cornell University* .

Karolyi, G. Andrew, and Ying Wu, 2018, A new partial-segmentation approach to modeling international stock returns, *Journal of Financial and Quantitative Analysis* 53, 507–546.

Kelly, Bryan, Seth Pruitt, and Yinan Su, 2019, Characteristics are covariances: A unified model of risk and return, *Journal of Financial Economics* 134, 501–524.

Kim, Soohun, Robert A. Korajczyk, and Andreas Neuhierl, 2019, Arbitrage portfolios, *Working paper, Northwestern University* .

Kim, Soohun, and Georgios Skoulakis, 2018, Ex-post risk premia estimation and asset pricing tests using large cross sections: The regression-calibration approach, *Journal of Econometrics* 204, 159–188.

Korajczyk, Robert, and Claude J. Viallet, 1989, An empirical investigation of international asset pricing, *Review of Financial Studies* 2, 553–585.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2018, Interpreting factor models, *Journal of Finance* 73, 1183–1223.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2020, Shrinking the cross section, *Journal of Financial Economics* 135, 271–292.

Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Contrarian investment extrapolation and risk, *Journal of Finance* 49, 1541–1578.

Lee, Kuan-Hui, 2011, The world price of liquidity risk, *Journal of Financial Economics* 99, 136–161.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2010, A skeptical appraisal of asset-pricing tests, *Journal of Financial Economics* 96, 175–194.

Lewis, Karen K., 2011, Global asset pricing, *Annual Review of Financial Economics* 3, 435–466.

Liew, Jim Kyung-Soo, and Maria Vassalou, 2000, Can book-to-market, size, and momentum be risk factors that predict economic growth?, *Journal of Financial Economics* 57, 221–245.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–3777.

Onatski, Alexei, 2010, Determining the number of factors from empirical distribution of eigenvalues, *Review of Economics and Statistics* 92, 1004–1016.

Pukthuanthong, Kuntara, and Richard Roll, 2009, Global market integration: an alternative measure and its application, *Journal of Financial Economics* 94, 214–232.

Pukthuanthong, Kuntara, Richard Roll, and Avanidhar Subrahmanyam, 2019, A protocol for factor identification, *Review of Financial Studies* 32, 1573–1607.

Raponi, Valentina, Cesare Robotti, and Paolo Zaffaroni, 2020, Testing beta-pricing models using large cross-sections, *Review of Financial Studies* 33, 2796–2842.

Rouwenhorst, K. Geert, 1999, Local return factors and turnover in emerging stock markets, *Journal of Finance* 54, 1439–1464.

Sentana, Enrique, 2002, Did the EMS reduce the cost of capital?, *The Economic Journal* 112, 786–809.

Sercu, Piet, 1980, A generalization of the international asset pricing model, *Revue de l'Association Francaise de Finance* 1, 91–135.

Shanken, Jay, 1982, The arbitrage pricing theory: Is it testable?, *Journal of Finance* 37, 1129–1140.

Shanken, Jay, 1985, Multivariate tests of the zero-beta CAPM, *Journal of Financial Economics* 14, 327–348.

Shanken, Jay, 1990, Intertemporal asset pricing: An empirical investigation, *Journal of Econometrics* 45, 99–120.

Shanken, Jay, 1992a, The current state of the arbitrage pricing theory, *Journal of Finance* 47, 1569–1574.

Shanken, Jay, 1992b, On the estimation of beta-pricing models, *The Review of Financial Studies* 5, 1–33.

Solnik, Bruno, 1974a, An equilibrium model of the international capital market, *Journal of Economic Theory* 8, 500–524.

Solnik, Bruno, 1974b, The international pricing of risk: An empirical investigation of the world capital market structure, *Journal of Finance* 29, 48–54.

Solnik, Bruno, 1983, International arbitrage pricing theory, *Journal of Finance* 38, 449–457.

Stehle, Richard, 1977, An empirical test of the alternative hypotheses of national and international pricing of risky assets, *Journal of Finance* 32, 493–502.

Stulz, René, 1981, A model of international asset pricing, *Journal of Financial Economics* 9, 383–406.

Titman, Sheridan, K.C. John Wei, and Feixue Xie, 2013, Market development and the asset growth effect: international evidence, *Journal of Financial and Quantitative Analysis* 48, 1405–1432.

Uppal, Raman, Paolo Zaffaroni, and Irinia Zviadadze, 2019, Correcting misspecified stochastic discount factors, *Working paper, EDHEC* .

Verdelhan, Adrien, 2018, The share of systematic risk in bilateral exchange rates, *Journal of Finance* 73, 375–418.

Watanabe, Akiko, Yan Xu, Tong Yao, and Tong Yu, 2013, The asset growth effect: Insights from international equity markets, *Journal of Financial Economics* 108, 529–563.

White, Halbert, 1982, Maximum likelihood estimation of misspecified models, *Econometrica* 50, 1–25.

## Figures and Tables

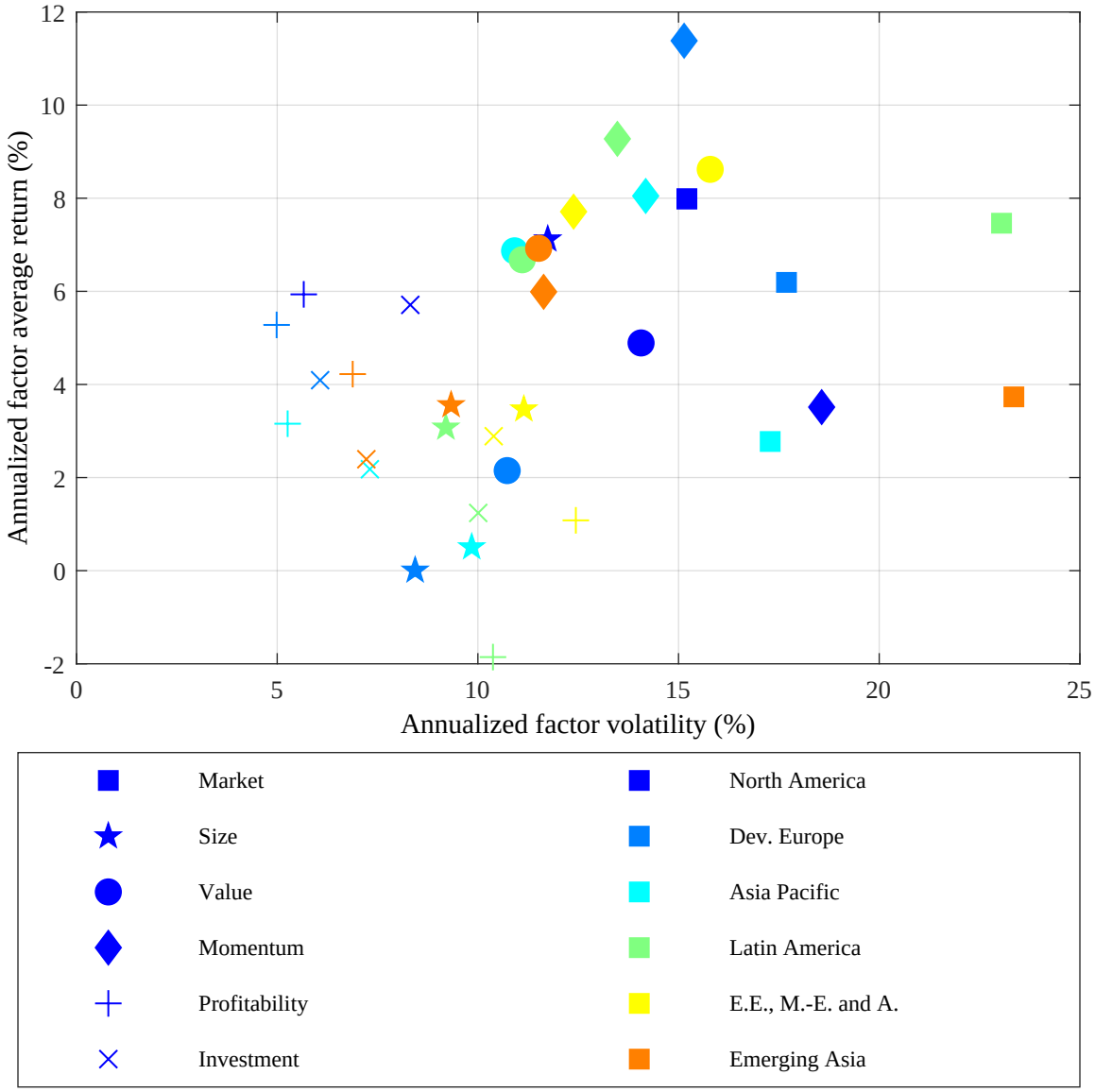


**Figure 1 Regional factor risk and return - 1996-2018**
We report regional factor average returns as a function of their volatility. We construct a market, size, value, momentum, profitability, and investment factor for each of our 47 countries. We build regional factors for North America, Developed Europe, Asia Pacific, Latin America, Emerging Europe, Middle East and Africa, and Emerging Asia. For each factor and region, we use the countries in this region and their lagged total market capitalization in USD to compute value-weighted returns. All returns are monthly, in USD, start in October 1996 when all regions are available, and end in October 2018.

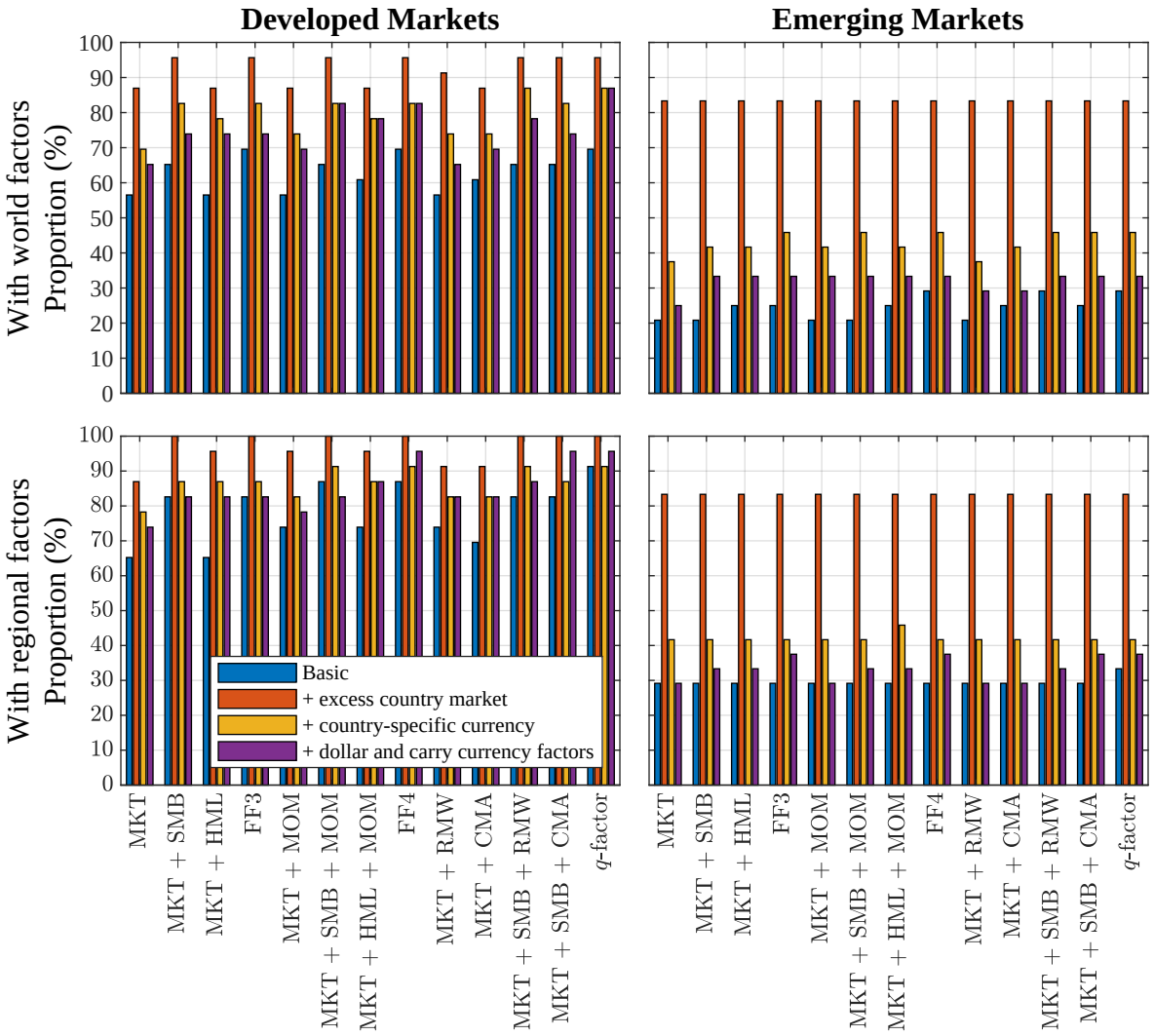


**Figure 2 Which models capture the factor structure?**
We report the proportion in % of countries for which the GOS2 diagnostic criterion is negative. The diagnostic criterion, given in Equation (A.10), checks for a remaining factor structure in the time-series of residuals. A positive value for the diagnostic criterion indicates that there remains at least one factor in the residuals obtained from the first step time-series regressions. A negative value says that the factors used in the asset pricing model capture the factor structure in stock returns. We report on Developed Markets in the left column and on Emerging Markets in the right column. The top (bottom) row uses factors aggregated at the world (regional) level. We test for different combinations of world and regional factors. For each combination, we report on the basic model, the model augmented with the excess country market factor, the model augmented with the country-specific currency, and the model augmented with the currency carry and dollar factors. All returns are monthly and in USD. We use a maximum condition number of 25, instead of the threshold of 15 used in our main model estimations, to ensure a sufficient number of stocks kept in models with a large number of factors.

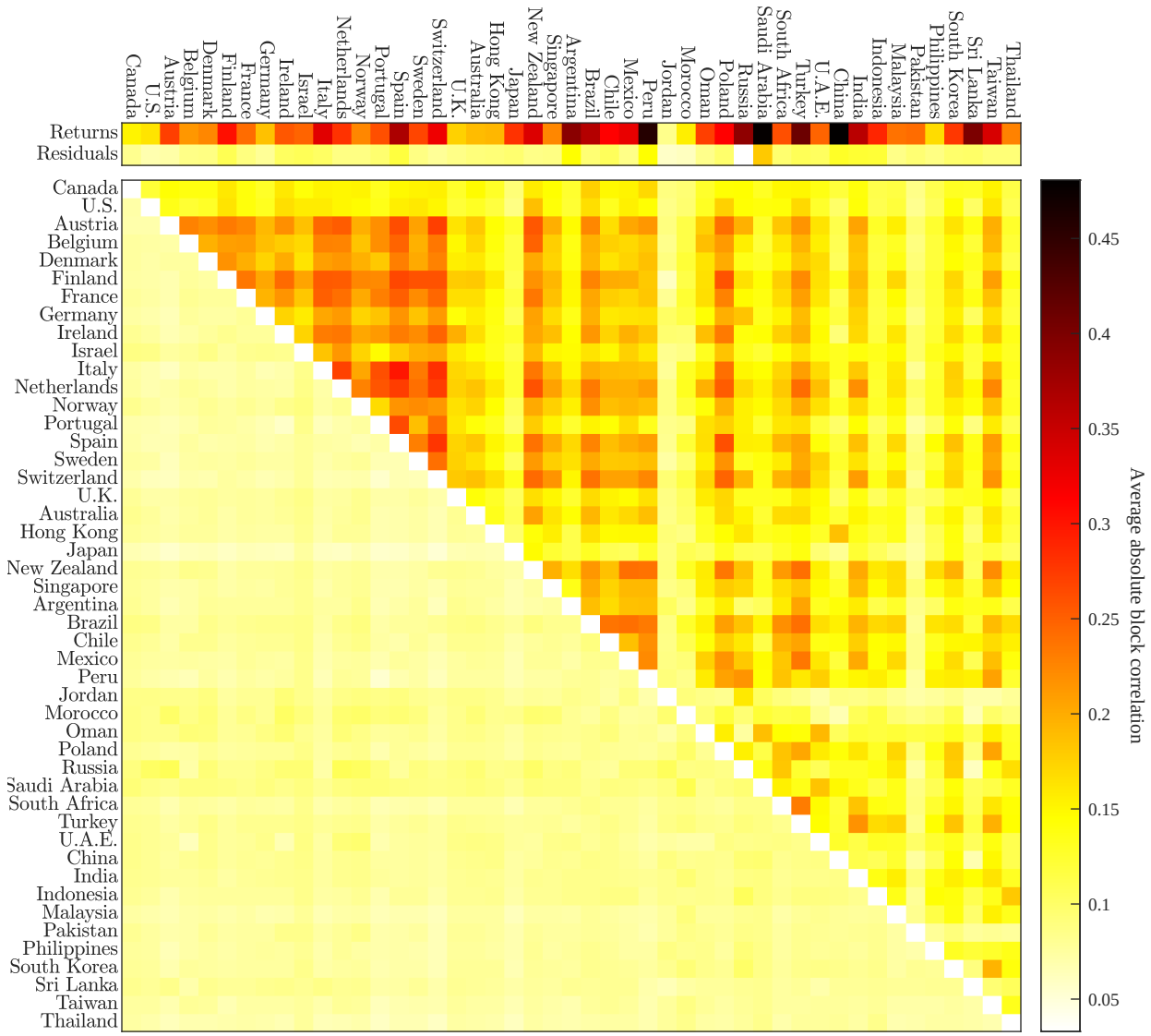


**Figure 3 Average absolute return and residual correlations**

We report the cross-sectional average of the absolute values of the correlations between individual stock returns and residuals from the mixed world four-factor model. The top panel reports on cross-sectional average of the absolute values of the correlations between stocks of the same country. In the bottom panel, we report cross-sectional average of the absolute values of the correlations between each stock in a country and each stock in another country. The elements above the diagonal report on cross-sectional average of the absolute values of the stock return correlation. The elements below the diagonal report on cross-sectional average of the absolute values of the residual correlation. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically. We only keep pairs of stocks for which we have more than 24 months to compute their correlation. All returns are monthly and in USD.

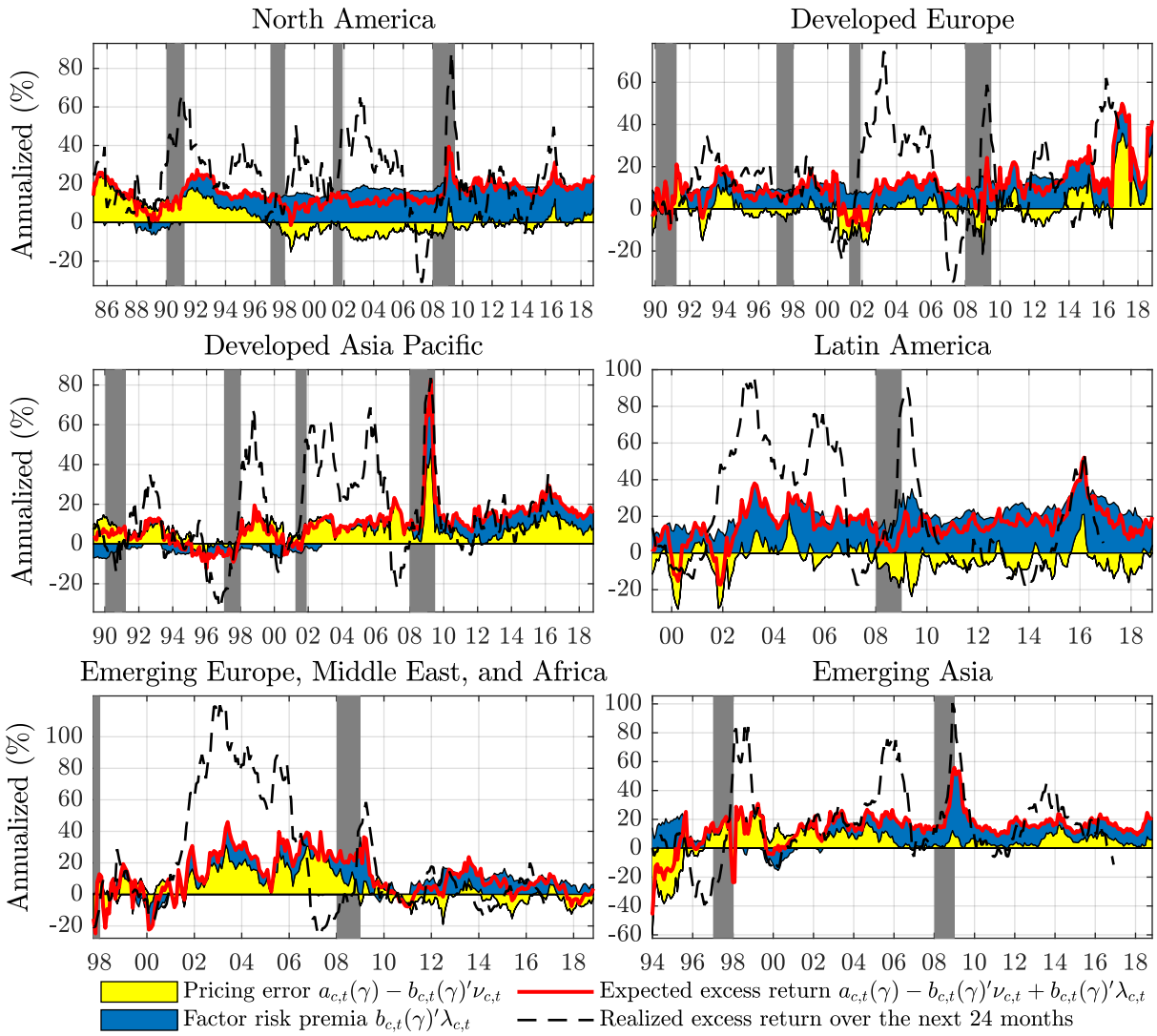


**Figure 4 Decomposition of expected returns in the mixed world four-factor**
Each month, we compute the pricing errors, $a_{c,t}(\gamma) - b_{c,t}(\gamma)'\nu_{c,t}$, and sum of factor risk premia, $b_{c,t}(\gamma)'\lambda_{c,t}$, across all available stocks. We report the equal-weighted average pricing error in yellow, factor risk premium in blue, and total expected return using a red line. We report using a dashed line the forward 24-month equal-weighted average excess return. We report results for each of the three DM regions and EM regions. All returns are monthly and in USD. We use recession dates from the NBER for the U.S. and the Economic Cycle Research Institute for non-U.S. countries. We build a recession indicator for each region, which is equal to one when at least half of the countries in the region are in a recession. We report in each figure gray areas to denote recession periods (across all DM regions for DMs and all EM regions for EMs).

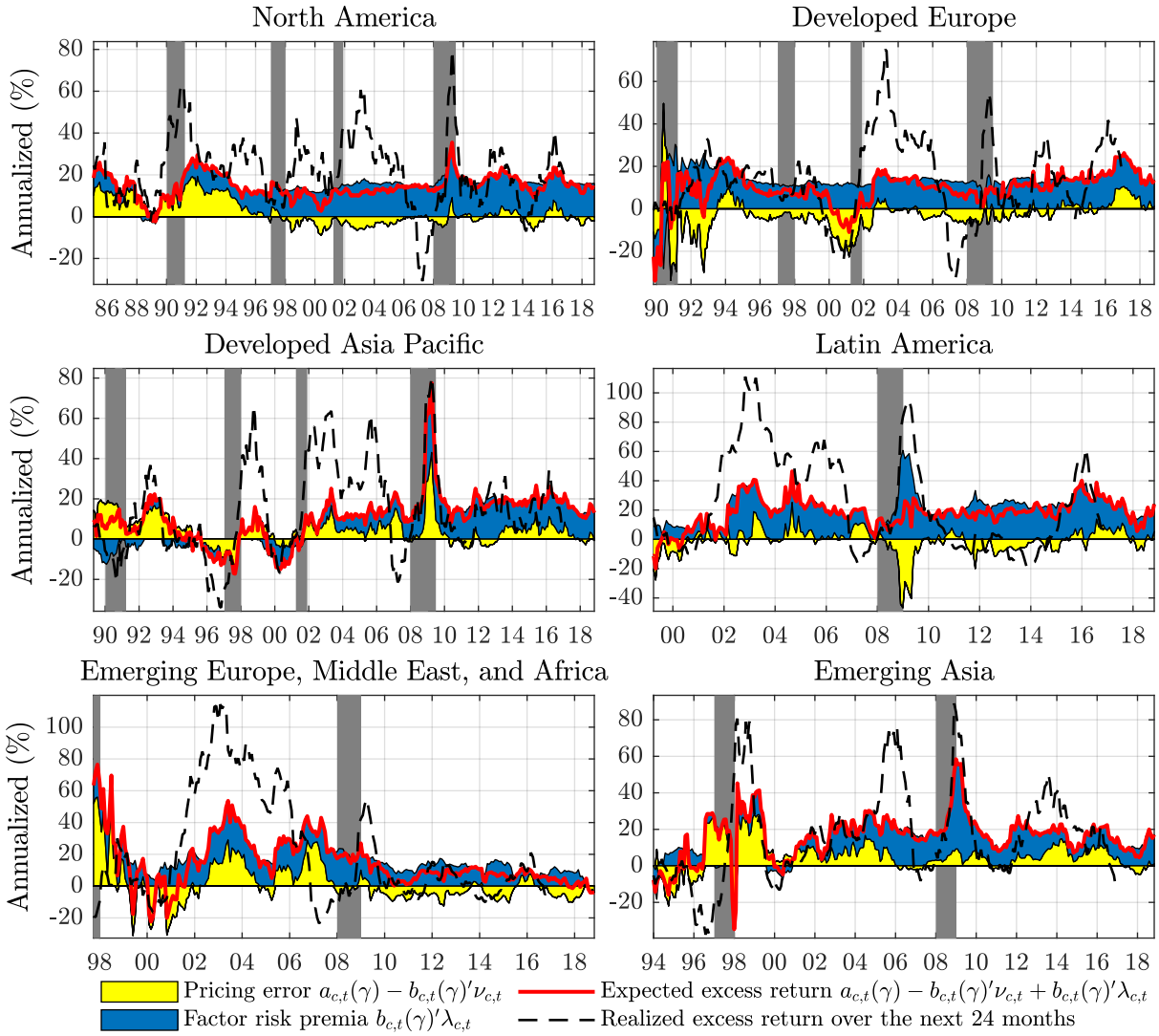


**Figure 5 Decomposition of expected returns in the mixed world $q$-factor**
Each month, we compute the pricing errors, $a_{c,t}(\gamma) - b_{c,t}(\gamma)'\nu_{c,t}$, and sum of factor risk premia, $b_{c,t}(\gamma)'\lambda_{c,t}$, across all available stocks. We report the equal-weighted average pricing error in yellow, factor risk premium in blue, and total expected return using a red line. We report using a dashed line the forward 24-month equal-weighted average excess return. We report results for each of the three DM regions and EM regions. All returns are monthly and in USD. We use recession dates from the NBER for the U.S. and the Economic Cycle Research Institute for non-U.S. countries. We build a recession indicator for each region, which is equal to one when at least half of the countries in the region are in a recession. We report in each figure gray areas to denote recession periods (across all DM regions for DMs and all EM regions for EMs).

## Table 1 Summary statistics across countries and regions

| Country/ Region | Start date (i) | Number of stocks (ii) | Annualized average return (%) Market (iii) | Size (iv) | Value (v) | Mom. (vi) | Prof. (vii) | Inv. (viii) | Annualized volatility (%) Market (ix) | Size (x) | Value (xi) | Mom. (xii) | Prof. (xiii) | Inv. (xiv) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Panel A: Summary statistics for 47 countries* | | | | | | | | | | | | | | |
| Argentina | Jan 99 | 158 | 6.67 | 9.23 | 14.33 | 6.51 | −3.06 | 4.34 | 34.52 | 23.91 | 28.86 | 26.69 | 24.60 | 26.02 |
| Australia | Jan 96 | 3,309 | 8.44 | −1.23 | 2.46 | 20.24 | 10.77 | 9.10 | 20.74 | 13.40 | 12.97 | 15.22 | 18.21 | 11.20 |
| Austria | Jul 93 | 179 | 6.76 | 1.63 | 4.55 | 6.98 | −2.33 | 5.66 | 21.27 | 13.62 | 16.43 | 19.39 | 14.87 | 14.53 |
| Belgium | Sep 93 | 297 | 7.78 | 0.90 | 1.33 | 12.11 | −0.12 | 1.94 | 18.50 | 11.54 | 14.19 | 17.02 | 13.61 | 12.90 |
| Brazil | Nov 01 | 292 | 18.10 | 3.45 | 5.34 | 15.24 | 0.17 | 3.60 | 32.73 | 18.35 | 17.60 | 22.30 | 20.22 | 19.44 |
| Canada | Jan 85 | 5,272 | 7.21 | −0.93 | −1.50 | 16.71 | 4.72 | 7.06 | 18.11 | 10.89 | 15.83 | 18.67 | 11.93 | 10.20 |
| Chile | Mar 99 | 269 | 10.79 | 5.20 | 4.74 | 8.46 | −5.62 | 1.21 | 20.07 | 11.23 | 12.73 | 13.74 | 14.27 | 10.95 |
| China | Jul 97 | 3,653 | 7.45 | 9.08 | 9.39 | −1.46 | 1.72 | 1.80 | 29.89 | 21.31 | 20.40 | 15.98 | 9.47 | 12.17 |
| Denmark | Dec 92 | 371 | 11.01 | −2.42 | −2.34 | 13.03 | 1.49 | 5.70 | 17.88 | 13.82 | 15.96 | 15.82 | 14.88 | 15.15 |
| Finland | Sep 93 | 265 | 12.09 | −3.08 | 1.22 | 12.32 | 2.36 | −0.58 | 27.44 | 14.42 | 22.84 | 21.00 | 17.54 | 18.07 |
| France | Jul 91 | 1,783 | 7.30 | 0.56 | 0.89 | 9.96 | 4.91 | 4.16 | 18.39 | 10.88 | 13.45 | 16.41 | 8.06 | 8.63 |
| Germany | Apr 91 | 1,304 | 6.38 | −4.61 | 4.13 | 12.69 | 3.06 | 5.21 | 19.45 | 10.91 | 14.56 | 18.60 | 10.80 | 12.29 |
| Hong Kong | Jul 00 | 2,140 | 7.06 | 0.28 | 8.04 | 10.02 | 6.48 | 4.21 | 21.03 | 16.68 | 12.59 | 17.84 | 12.10 | 10.52 |
| India | Oct 97 | 4,647 | 12.40 | 6.55 | 3.09 | 14.95 | 11.43 | 8.03 | 30.40 | 24.06 | 21.97 | 25.55 | 19.36 | 16.29 |
| Indonesia | Jul 93 | 715 | 9.39 | 5.21 | 10.33 | 9.79 | 5.65 | 3.20 | 38.27 | 25.59 | 37.29 | 27.31 | 21.38 | 20.26 |
| Ireland | Jan 97 | 114 | 5.43 | 2.42 | 0.41 | 10.13 | 9.16 | −2.71 | 28.34 | 21.06 | 27.02 | 38.07 | 22.28 | 26.19 |
| Israel | Jul 98 | 721 | 8.12 | 1.78 | −2.42 | 14.85 | 5.93 | 5.27 | 21.48 | 14.66 | 17.78 | 17.38 | 14.10 | 14.94 |
| Italy | Jan 92 | 629 | 4.66 | 0.58 | 1.17 | 8.06 | 5.38 | −0.24 | 23.35 | 13.21 | 16.60 | 18.36 | 11.78 | 15.97 |
| Japan | Apr 89 | 4,815 | 0.07 | 3.18 | 10.32 | 1.37 | 0.90 | 1.13 | 20.26 | 11.38 | 11.58 | 15.70 | 5.42 | 8.44 |
| Jordan | Jul 02 | 255 | 7.87 | −4.47 | −6.62 | 9.86 | 9.48 | −2.69 | 18.32 | 15.36 | 20.74 | 16.90 | 16.19 | 16.67 |
| Malaysia | Jul 99 | 1,326 | 6.72 | −2.50 | 4.13 | 9.54 | 1.71 | −3.96 | 17.86 | 11.30 | 9.84 | 13.28 | 8.83 | 7.83 |
| Mexico | Jul 93 | 230 | 8.85 | 1.91 | 5.23 | 12.10 | −3.29 | 3.57 | 27.44 | 13.09 | 18.09 | 19.67 | 16.30 | 16.45 |
| Morocco | Jul 06 | 97 | 6.02 | −1.63 | −3.68 | 10.31 | 7.57 | 4.45 | 15.09 | 15.25 | 17.12 | 15.36 | 14.81 | 14.06 |
| Netherlands | Jul 91 | 359 | 8.46 | −1.31 | 6.39 | 4.37 | 0.51 | 2.50 | 18.34 | 12.78 | 16.08 | 19.64 | 12.64 | 12.34 |
| New Zealand | Sep 03 | 246 | 11.26 | 0.36 | 0.71 | 16.67 | −3.72 | −4.05 | 19.07 | 9.14 | 11.93 | 11.27 | 12.28 | 10.44 |
| Norway | Jul 91 | 473 | 10.04 | 1.99 | 1.07 | 12.40 | 4.99 | 3.85 | 23.92 | 14.02 | 18.87 | 21.20 | 18.02 | 16.53 |
| Oman | Jan 05 | 117 | 4.83 | 5.67 | −1.63 | 4.22 | 1.02 | 6.86 | 17.50 | 12.76 | 20.16 | 16.54 | 17.65 | 17.74 |
| Pakistan | Jan 99 | 564 | 17.05 | 9.83 | 4.93 | 13.25 | 1.62 | 0.95 | 27.99 | 16.71 | 20.88 | 16.58 | 19.12 | 15.26 |
| Peru | Mar 02 | 160 | 18.49 | 12.51 | 8.02 | 10.88 | −1.03 | 6.24 | 23.84 | 21.74 | 33.47 | 32.19 | 31.78 | 21.38 |
| Philippines | Jul 98 | 315 | 10.36 | 4.71 | 21.39 | 1.25 | 0.75 | 0.66 | 25.71 | 18.12 | 27.15 | 29.73 | 25.42 | 21.09 |
| Poland | Jul 99 | 1,071 | 7.94 | 1.73 | 1.87 | 16.94 | 4.99 | 0.71 | 29.62 | 16.10 | 20.61 | 17.13 | 14.08 | 15.31 |
| Portugal | Feb 98 | 141 | 2.23 | −0.69 | 1.05 | 16.87 | −1.56 | 3.78 | 22.19 | 16.97 | 18.48 | 21.42 | 17.72 | 15.95 |
| Russia | Sep 06 | 573 | 3.50 | 4.86 | 11.47 | 8.23 | 8.50 | 2.58 | 32.11 | 21.10 | 32.52 | 19.69 | 14.03 | 14.11 |
| Saudi Arabia | Feb 06 | 199 | −0.47 | −1.29 | 7.36 | −1.86 | 1.35 | 1.01 | 25.98 | 16.28 | 16.90 | 15.26 | 18.48 | 15.36 |
| Singapore | Jul 01 | 926 | 9.46 | −2.00 | 8.83 | 10.05 | 4.57 | 1.82 | 20.89 | 11.59 | 11.34 | 17.00 | 9.94 | 10.15 |
| South Africa | Sep 97 | 1,000 | 8.03 | 2.31 | 4.96 | 16.53 | 4.07 | 5.68 | 27.37 | 13.79 | 19.15 | 16.77 | 18.73 | 14.43 |
| South Korea | Jul 95 | 2,984 | 5.84 | 1.02 | 9.47 | 7.99 | 5.19 | 4.29 | 34.11 | 17.82 | 19.81 | 22.48 | 11.65 | 13.81 |
| Spain | Jul 93 | 385 | 8.12 | −0.01 | 2.52 | 7.84 | −0.53 | −1.66 | 22.28 | 12.78 | 14.21 | 18.23 | 11.30 | 12.23 |
| Sri Lanka | Oct 03 | 334 | 8.84 | 7.49 | 10.00 | 11.84 | 3.94 | −1.00 | 23.72 | 14.75 | 18.12 | 13.98 | 14.79 | 15.17 |

| Country/ Region | Start date (i) | Number of stocks (ii) | Market (iii) | Size (iv) | Value (v) | Mom. (vi) | Prof. (vii) | Inv. (viii) | Market (ix) | Size (x) | Value (xi) | Mom. (xii) | Prof. (xiii) | Inv. (xiv) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | *Annualized average return (%)* | | | | | | *Annualized volatility (%)* | | | | | |
| *... continued* | | | | | | | | | | | | | | |
| Sweden | Jul 97 | 1,013 | 8.97 | −0.13 | 1.19 | 12.56 | 2.53 | 6.50 | 24.20 | 12.80 | 18.36 | 21.45 | 11.98 | 16.69 |
| Switzerland | Jul 91 | 436 | 8.57 | −0.81 | 0.13 | 9.34 | 3.02 | 6.80 | 16.38 | 10.95 | 13.19 | 16.34 | 12.11 | 10.62 |
| Taiwan | Jul 98 | 2,264 | 5.06 | 0.68 | 1.18 | 3.63 | 0.01 | −1.35 | 25.39 | 14.45 | 17.19 | 17.44 | 11.90 | 14.39 |
| Thailand | Jul 93 | 1,013 | 7.66 | −1.71 | 6.92 | 5.34 | 11.10 | 3.94 | 32.62 | 20.31 | 23.36 | 28.97 | 15.84 | 16.56 |
| Turkey | Jul 02 | 521 | 14.86 | 0.75 | 11.38 | 2.24 | −4.55 | −4.31 | 37.97 | 12.52 | 16.11 | 14.95 | 12.08 | 11.63 |
| U.A.E. | Apr 07 | 116 | 7.68 | 5.20 | 7.22 | 8.69 | 11.11 | −4.79 | 23.45 | 20.32 | 22.86 | 21.13 | 25.68 | 31.67 |
| U.K. | Oct 89 | 4,565 | 5.26 | 0.24 | 0.73 | 12.42 | 5.85 | 2.64 | 16.21 | 13.08 | 13.36 | 15.52 | 8.98 | 8.28 |
| U.S. | Jan 85 | 11,776 | 9.01 | 5.52 | 3.19 | 5.28 | 6.29 | 5.30 | 14.83 | 11.57 | 12.64 | 16.18 | 5.51 | 7.96 |
| *Panel B: Summary statistics across 47 countries* | | | | | | | | | | | | | | |
| Minimum | Jan 85 | 97 | −0.47 | −4.61 | −6.62 | −1.86 | −5.62 | −4.79 | 14.83 | 9.14 | 9.84 | 11.27 | 5.42 | 7.83 |
| Average | Mar 97 | 1,370 | 8.25 | 1.87 | 4.36 | 9.80 | 3.16 | 2.52 | 23.97 | 15.28 | 18.58 | 19.30 | 15.16 | 14.86 |
| Maximum | Apr 07 | 11,776 | 18.49 | 12.51 | 21.39 | 20.24 | 11.43 | 9.10 | 38.27 | 25.59 | 37.29 | 38.07 | 31.78 | 31.67 |
| *Panel C: Summary statistics for regions* | | | | | | | | | | | | | | |
| World | Jan 85 | 64,392 | 7.06 | 3.17 | 4.51 | 6.51 | 4.63 | 3.96 | 15.22 | 7.83 | 9.44 | 12.37 | 4.01 | 5.79 |
| Developed Mkts | Jan 85 | 41,519 | 7.14 | 3.06 | 4.31 | 6.45 | 4.66 | 4.02 | 15.04 | 8.26 | 9.91 | 13.13 | 4.15 | 6.03 |
| Emerging Mkts | Jul 93 | 22,070 | 4.92 | 3.35 | 6.57 | 6.95 | 5.11 | 1.72 | 21.73 | 7.73 | 9.48 | 9.47 | 7.44 | 6.25 |
| North America | Jan 85 | 17,048 | 8.84 | 5.10 | 2.90 | 6.06 | 6.19 | 5.40 | 14.84 | 11.19 | 12.43 | 15.94 | 5.34 | 7.77 |
| Dev. Europe | Oct 89 | 13,035 | 5.95 | −0.91 | 1.37 | 10.81 | 4.03 | 3.06 | 16.95 | 8.13 | 9.83 | 13.52 | 5.30 | 5.89 |
| Asia Pacific | Apr 89 | 11,436 | 1.26 | 1.78 | 8.80 | 4.38 | 2.59 | 1.73 | 19.47 | 10.00 | 10.16 | 13.76 | 5.25 | 7.22 |
| Latin America | Jul 93 | 1,109 | 7.85 | 3.81 | 7.32 | 10.83 | −4.02 | 3.42 | 23.57 | 10.62 | 13.40 | 13.38 | 12.00 | 13.92 |
| E.E., M.-E. and A. | Sep 97 | 3,480 | 7.22 | 3.47 | 8.62 | 7.71 | 1.08 | 2.89 | 26.18 | 11.15 | 15.79 | 12.38 | 12.44 | 10.39 |
| Emerging Asia | Jul 93 | 17,481 | 4.43 | 2.57 | 5.42 | 6.61 | 7.23 | 0.81 | 23.15 | 9.60 | 11.02 | 11.20 | 9.56 | 7.81 |

We report the start date, total number of stocks, and annualized average returns and volatilities for risk factors across countries and regions. The end date is October 2018. We construct a market, size, value, momentum, profitability, and investment long-short factor for each country, and form regional factors by value-weighting country factors using the lagged total market capitalization in U.S. dollars. Panel A reports on 47 countries. We present in Panel B the minima, averages, and maxima across the 47 countries. In Panel C, we report the summary statistics for each region. All returns are monthly and are in U.S. dollars. The market factor is in excess of the U.S. one-month T-bill rate.

**Table 2 Risk premium estimates in mixed world models**

| Factor | Region | Average risk premia per month (%) | | Proportion of significant risk premia (%) | |
|---|---|---|---|---|---|
| | | Four-factor *(i)* | $q$-factor *(ii)* | Four-factor *(iii)* | $q$-factor *(iv)* |
| Market | DM | 0.65 | 0.22 | 61 | 70 |
| | EM | 0.70 | 0.28 | 71 | 67 |
| Excess country market | DM | −0.08 | 0.29 | 43 | 83 |
| | EM | 1.44 | 0.59 | 54 | 50 |
| Size | DM | 0.27 | 0.49 | 74 | 83 |
| | EM | 0.74 | 0.07 | 75 | 63 |
| Value | DM | 0.13 | | 74 | |
| | EM | 1.11 | | 67 | |
| Momentum | DM | 0.19 | | 52 | |
| | EM | 0.21 | | 42 | |
| Profitability | DM | | 0.15 | | 61 |
| | EM | | 0.04 | | 50 |
| Investment | DM | | 0.23 | | 57 |
| | EM | | 0.22 | | 63 |

We report on the average risk premium estimates for each factor in columns *(i)* and *(ii)* and their significance at the 5% level in columns *(iii)* and *(iv)*. Because the common instruments, except the constant, are standardized to have a mean of zero, the average risk premium corresponds to the parameter for the constant, $\Lambda_0$. We report the average risk premium across DMs and EMs for the mixed world four-factor model in columns *(i)* and *(iii)* and for the mixed world $q$-factor model in columns *(ii)* and *(iv)*.

**Table 3 Which instruments drive time-variations in factor exposures in the mixed world four-factor model?**

| Factor | Region | Constant | Country dividend yield | Size | Value | Momentum |
|---|---|---|---|---|---|---|
| | | *(i)* | *(ii)* | *(iii)* | *(iv)* | *(v)* |
| Market | DM | 0.96 | 0.01 | −0.05 | −0.04 | 0.07 |
| | | (100.00) | (52.79) | (57.98) | (59.20) | (71.97) |
| | EM | 0.94 | 0.03 | 0.09 | 0.11 | 0.09 |
| | | (100.00) | (59.61) | (53.41) | (56.55) | (70.87) |
| Excess country market | DM | 0.97 | 0.04 | −0.16 | 0.18 | 0.07 |
| | | (100.00) | (61.60) | (63.41) | (64.31) | (78.44) |
| | EM | 1.06 | 0.00 | −0.04 | 0.19 | −0.02 |
| | | (100.00) | (59.93) | (55.69) | (58.18) | (76.46) |
| Size | DM | 0.80 | 0.02 | −1.26 | 0.39 | −0.15 |
| | | (100.00) | (43.08) | (57.36) | (60.12) | (72.76) |
| | EM | 0.23 | −0.07 | −0.63 | −0.02 | −0.27 |
| | | (100.00) | (56.14) | (58.06) | (58.87) | (69.65) |
| Value | DM | 0.10 | −0.09 | −0.89 | 1.10 | −0.23 |
| | | (100.00) | (30.62) | (39.29) | (45.09) | (47.71) |
| | EM | 0.00 | −0.17 | −0.38 | 0.35 | −0.13 |
| | | (100.00) | (35.25) | (35.25) | (34.80) | (41.17) |
| Momentum | DM | 0.12 | −0.04 | −0.40 | −0.34 | 0.44 |
| | | (100.00) | (27.55) | (38.79) | (41.02) | (52.51) |
| | EM | 0.04 | −0.05 | 0.22 | −0.17 | 0.02 |
| | | (100.00) | (38.72) | (38.60) | (39.53) | (47.65) |
| Median time-series $R^2$ (%) | DM | 22.48 | | | | |
| | EM | 37.22 | | | | |

We report the median coefficient value for each factor-instrument interaction in the time-series regressions for the mixed world four-factor model. For each factor in the first column, we report the median coefficient for each instrument in columns *(i)* to *(v)* across all stocks in developed markets and across all stocks in emerging markets. Below each coefficient median, we report in parentheses the proportion (in %) of stocks for which the regressor is selected by our methodology. Finally, we report in the last rows the median time-series regression $R^2$s.

**Table 4 Which instruments drive time-variations in factor exposures in the mixed world $q$-factor model?**

| Factor | Region | Constant | Country dividend yield | Size | Profitability | Investment |
|---|---|---|---|---|---|---|
| | | *(i)* | *(ii)* | *(iii)* | *(iv)* | *(v)* |
| Market | DM | 0.91 | −0.02 | −0.20 | −0.07 | 0.05 |
| | | (100.00) | (54.71) | (57.71) | (59.10) | (66.33) |
| | EM | 0.89 | 0.03 | −0.28 | 0.02 | −0.02 |
| | | (100.00) | (59.92) | (56.69) | (59.36) | (63.63) |
| Excess country market | DM | 0.90 | 0.05 | −0.46 | −0.00 | 0.04 |
| | | (100.00) | (61.13) | (62.55) | (63.31) | (72.72) |
| | EM | 1.03 | −0.01 | −0.25 | −0.03 | 0.02 |
| | | (100.00) | (60.42) | (56.97) | (62.57) | (68.51) |
| Size | DM | 0.68 | −0.02 | −1.58 | 0.06 | −0.06 |
| | | (100.00) | (55.13) | (61.13) | (61.07) | (69.67) |
| | EM | 0.15 | −0.09 | −1.23 | 0.15 | 0.01 |
| | | (100.00) | (62.68) | (61.90) | (64.10) | (67.98) |
| Profitability | DM | 0.04 | −0.15 | −0.06 | 0.22 | 0.15 |
| | | (100.00) | (58.92) | (60.21) | (63.20) | (68.94) |
| | EM | −0.35 | −0.12 | −0.66 | 0.37 | −0.02 |
| | | (100.00) | (63.88) | (61.96) | (66.69) | (71.07) |
| Investment | DM | 0.14 | −0.15 | −1.73 | 0.14 | −0.39 |
| | | (100.00) | (54.92) | (59.63) | (61.34) | (67.52) |
| | EM | −0.03 | −0.18 | −1.62 | −0.04 | 0.15 |
| | | (100.00) | (59.25) | (61.04) | (63.27) | (67.73) |
| Median time-series $R^2$ (%) | DM | 21.85 | | | | |
| | EM | 36.41 | | | | |

We report the median coefficient value for each factor-instrument interaction in the time-series regressions for the mixed world $q$-factor model. For each factor in the first column, we report the median coefficient for each instrument in columns *(i)* to *(v)* across all stocks in developed markets and across all stocks in emerging markets. Below each coefficient median, we report in parentheses the proportion (in %) of stocks for which the regressor is selected by our methodology. Finally, we report in the last rows the median time-series regression $R^2$s.