

# Asset allocation implications of illiquid assets

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GFRI – UniGe and SFI

November 10, 2021

## Abstract

In the wake of the global financial crisis, with low returns on traditional fixed income instruments, institutional investors seek alternative investment vehicles with similar characteristics. Illiquid assets, such as infrastructure and real estate, have become an increasingly important share of their investments. In this paper, we study the performance impact in term of diversification and return of introducing illiquid assets in an otherwise standard allocation.

**Keywords:** illiquid asset, asset allocation, mean-variance, constraints.

## Key takeaways

- Illiquid asset classes provide sizable gains in terms of performance when added to standard asset classes
- Performance gains are the highest for low volatility strategies
- Performances differ when using publicly listed and private indices

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# 1 Introduction

The goal of this paper is to quantify how a typical institutional investor can benefit from investing in illiquid asset classes. The focus is on infrastructure, real estate and private equity. We rely on the widespread Modern Portfolio Theory put forward by Markowitz (1952), also known as mean-variance portfolio allocation. This setting provides quantitative guidance to select assets so that the portfolio allocation is driven by a trade-off between reward (average return) and risk (variance). It is a formalization and extension of diversification benefits in investing (not putting all of your eggs in one basket), the idea that owning different kinds of financial assets is less risky than owning only one type. Its key insight is that an asset risk and return should not be assessed by itself, but by how it contributes to a portfolio overall risk and return. The goal is to choose the right combination of assets among which to optimally distribute one's nest eggs.

The outline of the paper is as follows. In Section 2, we describe our methodology before presenting the data we use in our empirical analysis for both standard asset classes and illiquid asset classes in Section 3. We report our empirical results in Section 4 before concluding in Section 5.

## 2 Methodology

Our methodology relies on a mean-variance approach for a European representative investor and several performance measures.

### 2.1 Optimal diversification: Mean-Variance approach

We use an asset allocation framework where the goal is to get the best award while simultaneously controlling the taken risk when investing in a portfolio of assets. We aim at maximizing the average return so that the variance does not exceed a variance level through an optimal asset selection. The variance measures the dispersion around the average, or how the returns are volatile around their average level. A plot of the maximal average return for each given level of risk is known as the mean-variance frontier. We often use the standard deviation or volatility which is the square root of the variance so that reward and risk have the same measurement unit (yearly reward in %).

We can perform the optimisation over the whole sample period (static or unconditional) or over successive time periods in a rolling fashion (time-varying or conditional). The former

analysis is suited to a long-term horizon investor with a passive (through-the-cycle) approach while the latter is suited to a short-term horizon investor with an active (point-in-time) approach.

Both approaches rely on the same inputs. We use average returns, and variances for each asset class we want to invest in. We also need to measure potential diversification benefits (or not) coming from the dependence (or not) between the returns. Absence of dependence or reverse dependence is better for diversification. We use correlations to measure the linear dependence between two asset returns. In the unconditional approach, we use the full sample period to compute them, while in the conditional approach, we use only the data relative to each successive time periods in the rolling window, for example data over rolling windows of 20 quarters for 3-month returns.

## **2.2 Representative investor**

In the dynamic setting, we benchmark the performance with respect to improving the performance of a base portfolio with a fixed structure: 50% in Equity, 20% in Government Bonds, 20% in Corporate Bonds, and 10% in Cash. It corresponds to a typical balanced allocation for an institutional investor, and we think of such an investor as a representative institutional investor. We investigate whether managing actively one fifth of the portfolio where illiquid assets are included while keeping fixed four fifth in the base portfolio helps us to improve on a strategy solely invested in the base portfolio. We have observed that departures from those chosen weights do not alter qualitatively the reported conclusions below. In some cases below, we also consider short-sale constraints (you can only sell stocks if you have them already in your portfolio), namely that the allocations can only be positive. We consider a representative investor which takes a European perspective and converts all gains and losses for each asset class in Euro.

## **2.3 Performance measures**

To analyse the characteristics of the different asset classes and investment strategies, we use the minimum return, the maximum return, the average return, the average return in excess of the risk free rate, the standard deviation, the skewness (asymmetry), the kurtosis (tail thickness), the Sharpe ratio (excess average return per unit of risk), and the alpha (additional return in excess of the returns given by investing in passive portfolios tracking market, size, momentum, investment and profitability risk premia).

## 3 Data

In this section, we elaborate on the choice of representative investable indices we use in this study. We first consider the standard assets classes, and then discuss our choices for illiquid asset classes. All returns are computed using quarterly data denominated in Euro, and are annualized for clarity.

### 3.1 Standard asset classes

#### 3.1.1 Stock Index

As a representative stock index, we use the MSCI ACWI Investable Market Net Total Return Index (labeled MSAINT in all figures and tables). This index captures all sources of equity returns in 23 developed and 24 emerging markets. It covers a wide range of securities (9,000) from large to small market capitalisations, and across different style and sector segments.

#### 3.1.2 Government and Corporate Bond Indices

As a representative government bond index, we use the JP Morgan Government Bond Index EMU (labeled JPMGBE in all figures and tables) index which consists of approximately 300 bonds and 35 indices, primarily JPM Economic and Monetary Union indices. As representative corporate bond index, we use the Citi World Big excluding MBS hedged index (labeled WBEM-H in all figures and tables).

#### 3.1.3 Cash Index

Finally, as a representative cash investment, we use the Euro Cash Index LIBOR Total Return with 3 Month maturity (labeled ECLIT3 in all figures and tables).

#### 3.1.4 Summary statistics

We list the summary statistics for the four standard asset classes for the sample period 1999-03-31 to 2017-09-30 in Table 1. As expected, the stock index has both the highest return and volatility, and displays the largest drawdown. The government and corporate bond indices have similar characteristics with slightly higher volatility for the government bond index. The cash index provides the safest investment opportunity, albeit with the lowest average return.

**Table 1:** Summary statistics for the 4 standard asset classes. The sample period is 1999-03-31 to 2017-09-30. All returns are denominated in EUR.

	ECLIT3	JPMGBE	MSAINT	WBEM-H
Minimum	-0.10%	-5.49%	-21.90%	-3.10%
Maximum	1.38%	6.03%	26.00%	5.30%
Mean (an. % )	2.03%	4.59%	7.10%	4.30%
Stand. dev. (an. % )	0.87%	4.24%	17.80%	3.30%
Skewness	0.242	-0.436	-0.554	-0.134
Kurtosis	1.833	3.290	3.738	2.973
Q[0.01]	-0.09%	-4.10%	-21.70%	-2.80%
Q[0.99]	1.30%	5.22%	19.00%	4.90%
First order autocorrelation	0.949	-0.00051	0.102	0.042

## 3.2 Illiquid asset classes

For the illiquid asset classes, we distinguish below whether we use public indices (listed, market-valuation based) or private indices (appraisal based). Private indices are prone to return smoothing due to sluggish adjustments of valuations to new incoming information, or reporting delay issues. Hereafter we use the desmoothing technique described in Trojani, Orlowski, Popescu and Sali (2017) to better reflect the trade-offs of investment in the illiquid asset classes. We can view the results reported hereafter as conservative in the sense that desmoothing corrects for the dampening of the variability in the sample path artificially introduced by return smoothing. It better reflects the true return volatility (risk) exhibited by illiquid asset classes.

### 3.2.1 Infrastructure

As representative for the Infrastructure asset class, we use the indices published by MSCI. We use a Global index (MSCI ACWI Infrastructure), a European index (MSCI ACWI Infrastructure EMU), and an Emerging Market index (MSCI ACWI Infrastructure Emerging), which are public indices. We label them Inf Global-pub, Inf EMU-pub, and Inf EM-pub in the tables and figures.

### 3.2.2 Private Equity

As representative for the Private Equity asset class, we use a Global index published by Cambridge Associates (CA Global Buyout & Growth Equity), which is a private index. We label it PE Global in the tables and figures.

### 3.2.3 Real estate

As representative for the Real Estate asset class, we use a Global index published by MSCI (MSCI ACWI Core Real Estate), which is a public index. We also use a Global index published by Cambridge Associates (CA Real Estate), which is a private index. We label them RE Global public and RE Global private in the tables and figures.

### 3.2.4 Hedge fund

As representative for the Hedge Fund asset class, we use the indices published by Barclayhedge and Eurekahedge. We use a Global index (Barclay Hedge Fund Index), and a European index (Eurekahedge European). Both are private indices. We label them HF Global-priv and HF EMU-priv in the tables and figures.

### 3.2.5 Summary statistics

Table 2 provides summary statistics for the global region for both public and private illiquid assets.

**Table 2:** Summary statistics for the illiquid assets for the Global region. The sample period is 1999-03-31 to 2016-09-30 for RE pub and HF priv, and 1999-03-31 to 2017-03-31 for PE priv and RE priv, and 1999-06-30 to 2016-06-30 Inf pub. All returns are denominated in EUR.

Measure	HF priv	PE priv	RE priv	RE pub	Inf pub
Minimum	-12.30%	-21.40%	-37.80%	-31.00%	-23.30%
Maximum	24.40%	30.60%	24.90%	33.00%	31.80%
Mean (an. %)	8.70%	13.50%	9.70%	11.00%	4.50%
Stand. dev. (an. %)	13.40%	16.20%	17.50%	20.00%	17.50%
Skewness	0.384	-0.069	-1.09	-0.33	-0.187
Kurtosis	3.947	4.771	7.995	4.28	4.788
Q[0.01]	-12.30%	-17.70%	-19.50%	-21.00%	-20.40%
Q[0.99]	18.70%	21.20%	21.40%	22.00%	19.60%
First order autocorrelation	0.011	0.017	-0.064	0.11	0.049

Each asset class has roughly the same characteristics in terms of expected return. The real estate class is more volatile, exhibits more extreme positive or negative movements, and a higher probability of downturns.

## 4 Empirical Results

We start our empirical analysis by checking correlations between the different asset classes to see whether we can expect some diversification benefits from investing in the illiquid asset classes. Then we check the added value of such an investment before investigating potential differences across regions. We conclude by a further detailed analysis of the PrEQIn investable indices.

### 4.1 Correlation analysis between liquid and illiquid assets

Any improvement in performance, in the mean-variance framework, significantly relies on the (not too large) correlation of the additional assets with the base assets. We first look at unconditional correlation over the entire data sample and then focus our attention on rolling-window correlations. We face the natural tradeoff between saliency and precision, recent data is representative of more salient features of the dependence dynamics but considering longer estimation windows increases the precision of the estimated correlations. We therefore consider different rolling-window sizes for the time-varying correlation computation.

#### 4.1.1 Unconditional correlation

Figure 1 displays the unconditional correlation level for the entire sample, considering the base asset classes and the two publicly traded assets available for the Global region, namely the RE Global Public and Infrastructure Global Public. We can see that correlation between the illiquid assets and the global stock index are high, with 0.78 for the real estate index and 0.85 for the infrastructure index. We therefore expect the contribution of these two new asset classes to be limited in terms of diversification when considering the whole sample period. It is important to note however that the average return of 11% for the real estate index might make it an asset class to consider. Illiquid asset classes are negatively correlated with standard interest rates products, and are thus good diversifier tools with respect to them. The two illiquid asset classes are not redundant since they are only correlated at .58.

Figure 2 displays the unconditional correlation level for the entire sample, considering the base asset classes and the 3 privately traded assets available for the Global region, namely HF private, PE private and RE Global Private. We do not observe large differences when compared to the numbers reported for the publicly traded assets, and therefore the same comments apply. The only difference is the medium correlation we get between the global stock index and the private real estate index.

	ECLIT3	JPMGBE	MSAINT	WBEM-H	RE Global - pub	Inf Global - pub
ECLIT3	1.00	0.03	-0.36	0.16	-0.32	-0.31
JPMGBE	0.03	1.00	-0.29	0.88	-0.04	-0.17
MSAINT	-0.36	-0.29	1.00	-0.41	0.78	0.85
WBEM-H	0.16	0.88	-0.41	1.00	-0.10	-0.21
RE Global - pub	-0.32	-0.04	0.78	-0.10	1.00	0.58
Inf Global - pub	-0.31	-0.17	0.85	-0.21	0.58	1.00

**Figure 1:** Unconditional correlation matrix for the sample period 1999-06-30 to 2016-09-30. The data used is MSAINT, JPMGBE, WBEM-H and ECLIT3 for the base assets and RE Global Public and Infrastructure Global Public for the publicly traded illiquid assets.

	ECLIT3	JPMGBE	MSAINT	WBEM-H	HF priv	PE priv	RE priv
ECLIT3	1.00	0.03	-0.36	0.16	-0.12	-0.28	-0.26
JPMGBE	0.03	1.00	-0.29	0.88	-0.21	-0.36	-0.23
MSAINT	-0.36	-0.29	1.00	-0.41	0.80	0.84	0.46
WBEM-H	0.16	0.88	-0.41	1.00	-0.30	-0.44	-0.27
HF priv	-0.12	-0.21	0.80	-0.30	1.00	0.82	0.51
PE priv	-0.28	-0.36	0.84	-0.44	0.82	1.00	0.71
RE priv	-0.26	-0.23	0.46	-0.27	0.51	0.71	1.00

**Figure 2:** Unconditional correlation matrix for the sample period 1999-03-31 to 2016-09-30. The data used is MSAINT, JPMGBE, WBEM-H and ECLIT3 for the base assets and RE Global Public and Infrastructure Global Public for the publicly traded illiquid assets.

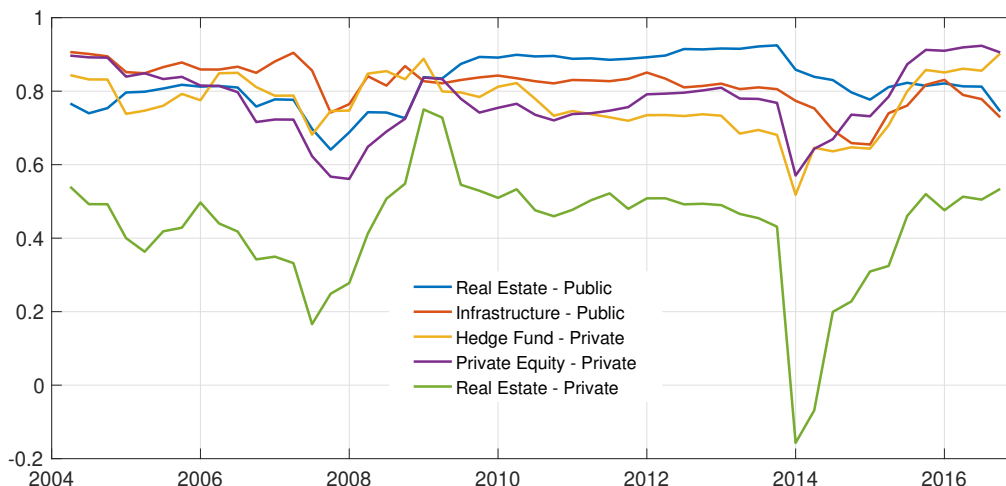
#### 4.1.2 Conditional correlation

It might be advantageous to adjust the asset allocation of our investment strategies based on temporary shifts in correlations. To illustrate these conditions, in Figure 3 we look at the evolution of the correlations between our main stock index (MSAINT) and the 5 illiquid asset classes (2 publicly traded: RE Global Public and Infrastructure Global Public, and 3 privately traded: HF private, PE private and RE Global Private). The correlations are high and rather stable except with the private real estate index. The latter is much lower and exhibits large swings through time, and becomes even negative in 2014.

## 4.2 Asset allocation performance: any added value?

We split the performance analysis in unconditional and conditional assessments. The former gives a broad overview of the potential performance improvement on the long-run, while the latter corresponds to achievable performance based on available information at the time of portfolio construction. The unconditional analysis is a "through-the-cycle" approach where we do not take into account the current particular state of the economy to choose the asset





**Figure 3:** Conditional correlations with the global stock index (MSAINT) for the sample period 2004-03-31 to 2016-09-30. The data used is RE Global Public and Infrastructure Global Public for the publicly traded illiquid assets and HF private, PE private and RE Global Private for the privately traded illiquid assets. The rolling window is 20 quarters.

allocation, while the conditional analysis is a "point-in-time" approach where we use the information of the current state of the economy to make the portfolio selection. In both cases, the analysis is performed for three investment regions, namely Global, Euro zone, and Emerging.

#### 4.2.1 Unconditional performance

We first consider the unconditional performance improvement of introducing illiquid assets. Focusing on the risk-return tradeoff, we compare the mean-variance efficient frontiers over the entire available dataset, with and without illiquid assets. Naturally, we expect a translation of the efficient frontier toward the North-West following the introduction of illiquid assets (which expands automatically the investment opportunity set), but this exercise allows us to gauge the magnitude of the improvement, as well as the marginal contribution of public and private illiquid asset indices. We display the efficient frontier in terms of standard deviation (volatility) instead of variance since it is easier to interpret because of sharing the same unit with returns.

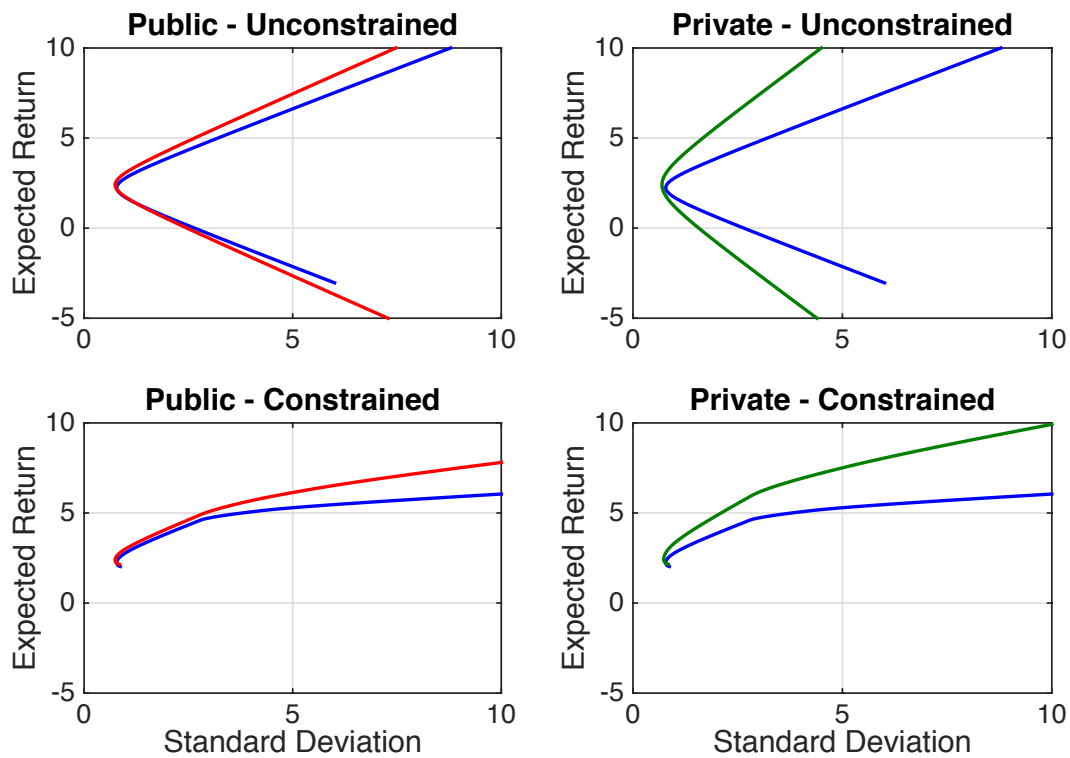
A volatility of 5% means that, according to the 68–95–99.7 rule, we have roughly 68%, 95% and 99.7% chances to have variations of 5%, 10% and 15% around the expected return.

Figure (4) displays the incremental contribution in terms of risk and return of the in-

introduction of the illiquid asset classes. Adding the two publicly traded asset classes Infrastructure Global and RE Global, has a marginal impact, whereas the improvement is significant for the introduction of the private investments HF global, RE global and PE global. For example, a portfolio with expected return of 5% has a yearly standard deviation of approximately 3% in the base investment universe and decreases to approximately 1.5% when allowing investments in private illiquid assets. Similarly, for a volatility of 5%, there is only a marginal gain in terms of expected return for the publicly traded asset classes. We only enjoy an additional 1%. On the contrary, we have a vertical shift of close to 5% for the privately traded asset classes. Adding illiquid asset classes does not seem to help if we seek a very conservative strategy. The global minimum variance portfolio (the GMVP is the most left point on the efficient frontier), which achieves the lowest volatility possible within the asset class mix, remains unaffected by the introduction of the illiquid asset classes, both for the publicly and privately traded cases. The previous comments carry over if we face short sales constraints (positive allocations only). To summarize, if we are willing to take a bit of risk and choose portfolios with higher volatilities than the one of the GMVP, it pays a lot (a bit) to consider the privately (publicly) traded asset classes as diversifier tools.

### 4.3 Conditional performance

In a time-varying setting, we could study a large number of possible configurations to assess the performance improvement based on the introduction of illiquid assets. First, we could consider an unconstrained allocation which would seek to maximize the Sharpe Ratio or minimize the variance through active rebalancing of the whole portfolio. This type of portfolio typically contains very large short and long positions, making it rather unrealistic. Selling and buying huge quantities is costly and sometimes even infeasible in practice, for example, because of limited short sales capacities. In order to maintain reasonable portfolio weights, we first consider a base portfolio with the following fixed structure: 50% in MSAINT, 20% in JPMGBE, 20% in WBEM-H, and 10% in ECLIT3. As already mentioned, it corresponds to a typical balanced allocation for an institutional investor. We then consider two dynamic strategies where we fix an 80 % allocation in the base portfolio, add the illiquid assets and optimize on the remaining 20% by either maximizing the Sharpe Ratio (Tangency) or minimizing the variance (GMVP), in both cases we impose short sales constraints. Such an approach is of a core-satellite type where the core is a balanced portfolio and the satellite is made of illiquid assets. We use a 5 years rolling window to estimate the conditional covariance matrix and expected returns, while the portfolio is rebalanced every quarter according



**Figure 4:** Efficient frontiers for the sample period 1999-03-31 to 2016-09-30. The frontier in blue is the basic stock bond cash allocation using MSAINT, JPMGBE, WBEM-H and ECLIT3. The frontier in red adds the publicly traded illiquid assets Infrastructure Global and RE Global to the base investment universe. The frontier in green adds the privately traded illiquid assets HF global, RE global, PE global to the base investment universe. The top panels are unconstrained and the bottom panels are short sales constrained.

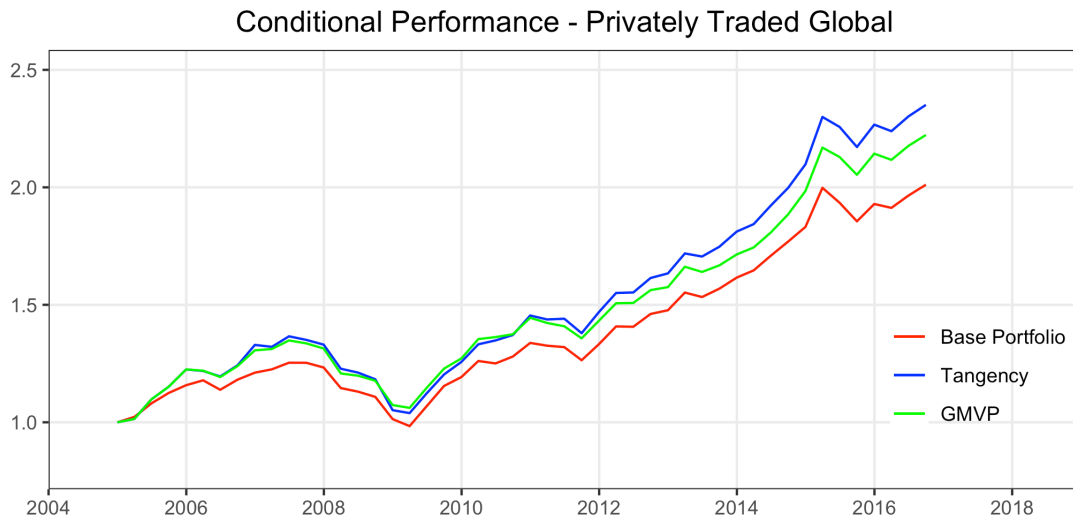
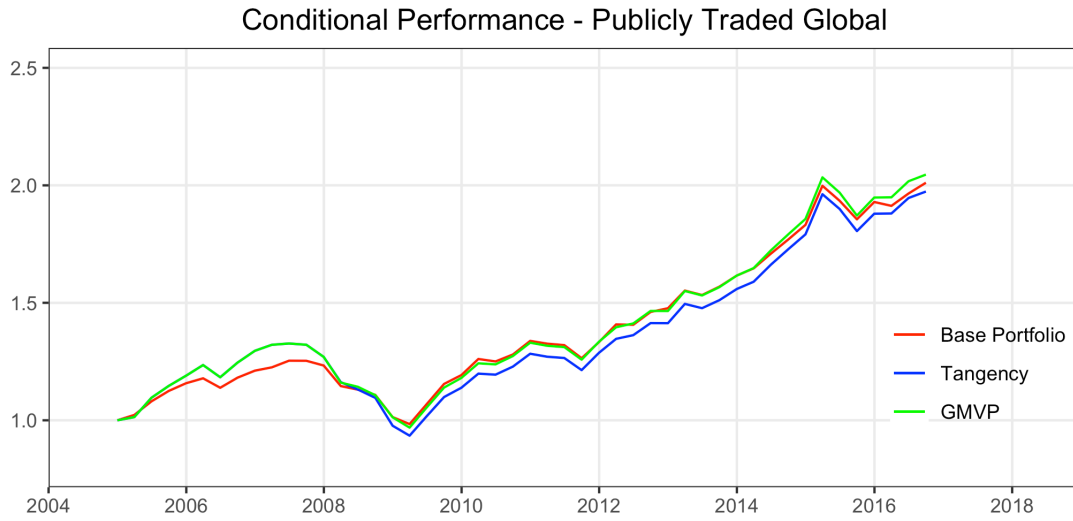
to the updated weights optimised with the information collected on the previous 5 years. We repeat this exercise for the four available regions, splitting between publicly and privately traded illiquid assets indices.

### 4.3.1 Conditional performance - global region

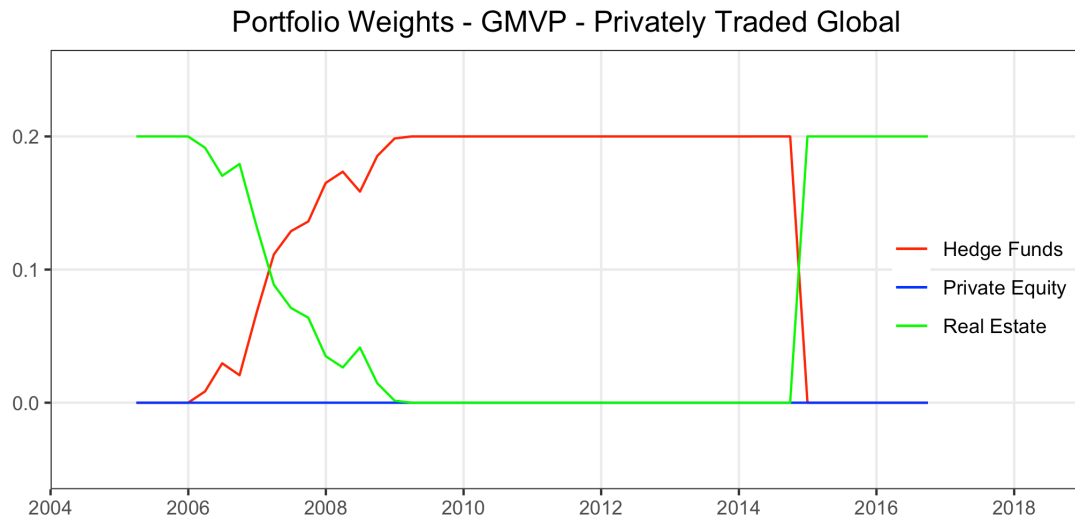
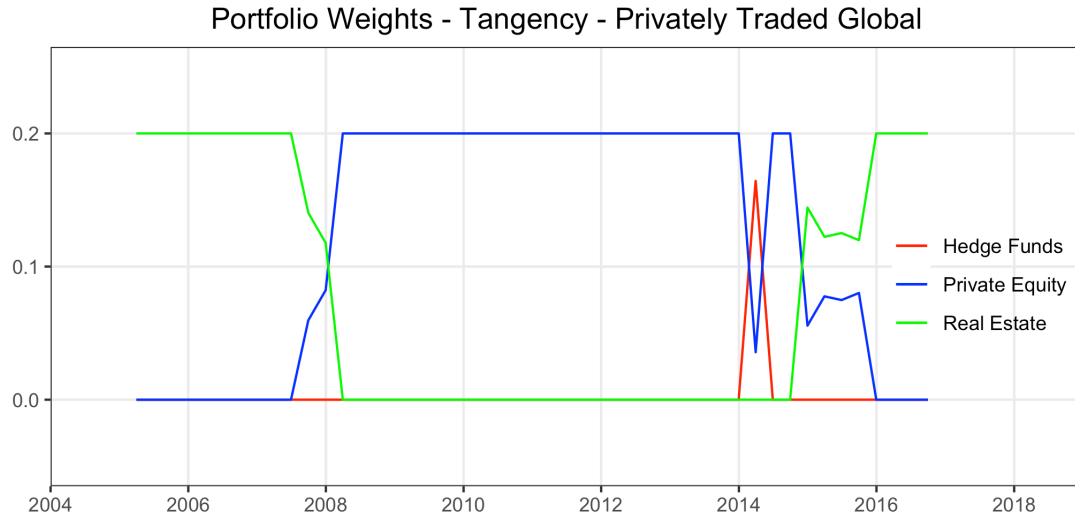
Figure 5 shows that the improvement in performance is the largest by considering the introduction of the privately traded indices, for both the Tangency and GMVP strategies. While, in the unconditional case, it is not really beneficial for the Tangency to go for additional investment in the illiquid asset classes. It does not apply for a conditional analysis since the GMVP performs much better in the presence of the illiquid asset classes, and that for both the publicly and privately traded cases. The Tangency strategy requires as statistical inputs both estimated means and estimated covariance matrix. The GMVP strategy only requires the latter. Estimated expected returns are often noisy estimates because of small sample sizes. They are also much less persistent than estimated variances and are more difficult to estimate in a stable way. It might explain the lower performance of the Tangency strategy when compared to the GMVP strategy using the public investment series.

Figure 6, which displays the invested weights for the Tangency and GMVP strategies for the privately traded indices, further indicates the source of this additional performance. For both strategies, the investment in illiquid assets is initially concentrated in the Real Estate sector until 2008. Then the two strategies branch out with the Tangency portfolio shifting to Private Equity until 2014 and GMVP portfolio shifting to Hedge Funds until 2015. The Tangency portfolio oscillated between the three illiquid asset classes until 2016 and finally both strategies move back into the Real Estate sector. To summarize, the introduction of the illiquid asset classes delivers additional performance for both conservative and more aggressive time-varying allocations.

Table 3 provides detailed summary statistics on the portfolio respective performance. The increase in Sharpe Ratio using the private investment series is due to a combination of a sizable increase in average return and a moderate increase in standard deviation. The diversification effect is most visible in the GMVP strategy using the private investment series, where the standard deviation is comparable to the base strategy (7.57% for the base strategy and 7.90% for the GMVP) while the average return increases from 6.27% to 7.16%. The best performance in terms of Sharpe Ratio is attained by the Tangency portfolio using the private investment series (increase from 0.59 for the base strategy to 0.69 for the latter). The GMVP strategy has also a high Sharpe Ratio of 0.67. All investment strategies generate



**Figure 5:** Conditional performance for the sample period 2005-03-31 to 2016-09-30. The base portfolio composed of 50% in MSAINT, 20% in JPMGBE, 20% in WBEM-H and 10% in ECLIT3 is in red. The top panel adds the publicly traded illiquid assets Infrastructure Global and RE Global. The bottom panel adds the privately traded illiquid assets HF global, RE global, PE global. We consider the tangency (in blue) and GMVP (in green) portfolios.



**Figure 6:** Portfolio weights for the sample period 2005-03-31 to 2016-09-30 for the privately traded indices. The top panel considers the Tangency portfolio and the bottom panel considers the GMVP portfolio.

negatively skewed returns and similar level of kurtosis (extreme event or tail risk measure), they all have comparable drawdowns as well.

**Table 3:** Summary statistics for the conditional portfolio performance. The sample period is 2005-03-31 to 2016-09-30. All returns are denominated in EUR. When indicated with (an. %) returns are annualized in percentage terms. \*\*\*, \*\* and \* indicate statistical significance at the 1%, 5% and 10% levels.

	Base	TG - pub	GMVP - pub	TG - priv	GMVP - priv
Minimum	-0.085	-0.108	-0.086	-0.111	-0.088
Maximum	0.091	0.095	0.095	0.096	0.093
Mean (an. %)	6.27	6.20	6.49	7.69	7.16
Excess Mean (an. %)	4.68	4.61	4.90	6.10	5.57
Stand. dev. (an. %)	7.57	8.73	8.43	8.40	7.90
Skewness	-0.44	-0.61	-0.47	-0.66	-0.43
Kurtosis	3.16	3.27	2.87	3.67	3.13
Sharpe ratio (an. %)	0.59	0.51	0.56	0.69	0.67
Alpha (FF-5, an. %)	0.88	0.28	0.49	2.22**	1.75*

#### 4.4 Performance across regions

In this section, we analyze the incremental contribution of the publicly traded illiquid assets<sup>1</sup> for different regions, namely Global, Europe and Emerging, and we also consider two different strategies, one that maximizes the Sharpe Ratio (Tangency) and one that minimizes the variance (GMVP).

Figure 7 displays the performance of the strategies. It clearly appears that including illiquid assets from Emerging markets significantly outperforms other regional choices. Also, the GMVP strategy dominates the Tangency strategy for all regions. The numbers reported in Table 4 corroborate these findings. To summarize, the introduction of the illiquid asset classes delivers additional performance for all regions but especially for conservative allocations in Emerging markets.

<sup>1</sup>We focus this analysis on publicly traded assets since data for privately traded assets is not available for all regions. Given the results of the previous sections for the privately traded assets, we expect the performance gains to be even higher when we add them.

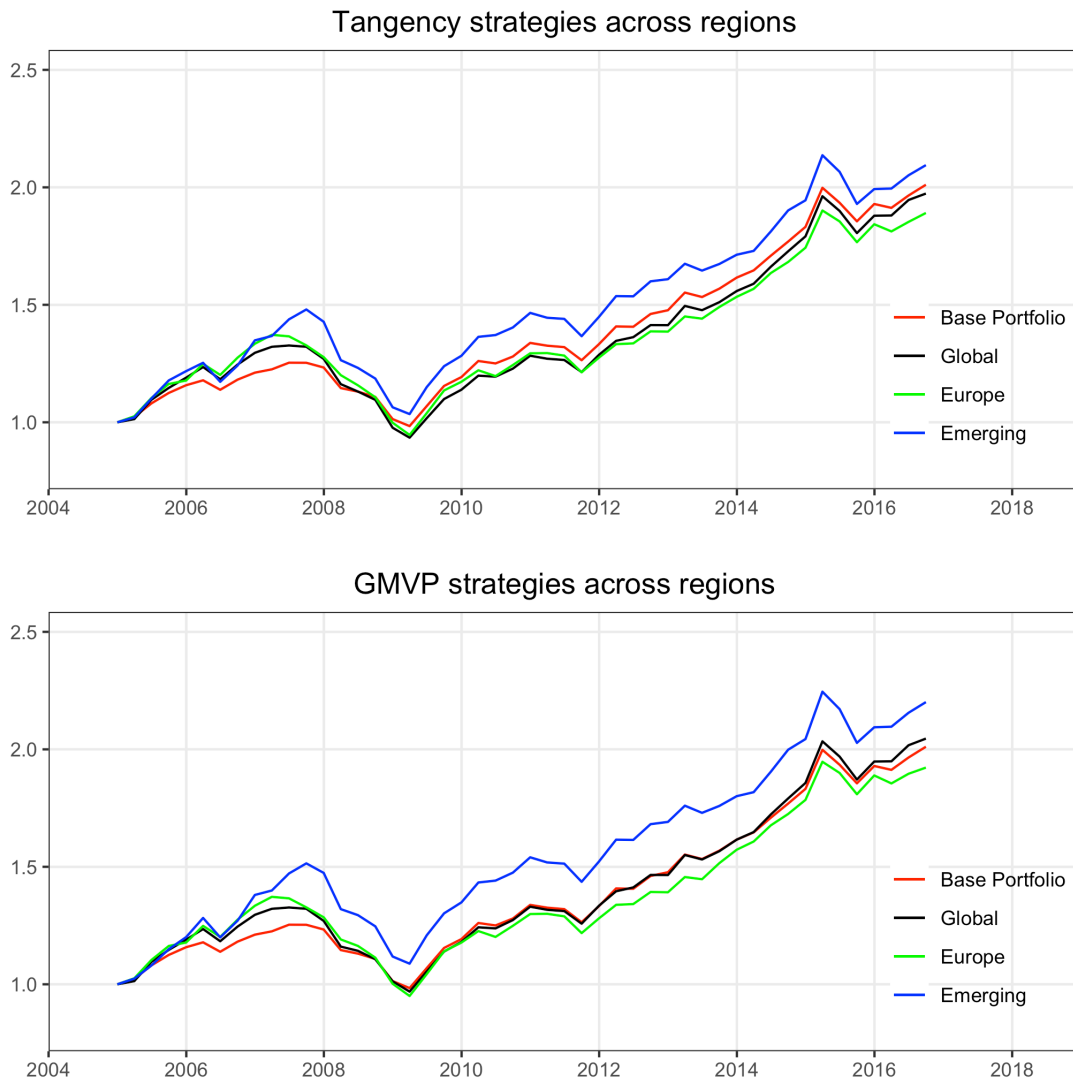
**Table 4:** Summary statistics for the conditional portfolio performance. The sample period is 2005-03-31 to 2016-06-30. All returns are denominated in EUR. When indicated with (an. %) returns are annualized in percentage terms. \*\*\*, \*\* and \* indicate statistical significance at the 1%, 5% and 10% levels.

	TG			GMVP		
	Global	EUR	EM.	Global	EUR.	EM.
Min	-0.108	-0.098	-0.114	-0.086	-0.100	-0.105
Max	0.095	0.098	0.111	0.095	0.098	0.111
Mean (an. %)	6.20	5.84	6.81	6.49	5.99	7.23
Excess Mean (an. %)	4.61	4.25	5.23	4.90	4.40	5.64
Stand. dev. (an. %)	8.73	8.82	9.78	8.43	8.91	9.61
Skewness	-0.61	-0.33	-0.62	-0.47	-0.39	-0.62
Kurtosis	3.27	2.63	3.31	2.87	2.72	3.22
Sharpe ratio (an. %)	0.51	0.46	0.52	0.56	0.47	0.57
Alpha (FF-5f, an. %)	0.28	0.52	1.26	0.49	0.49	2.12

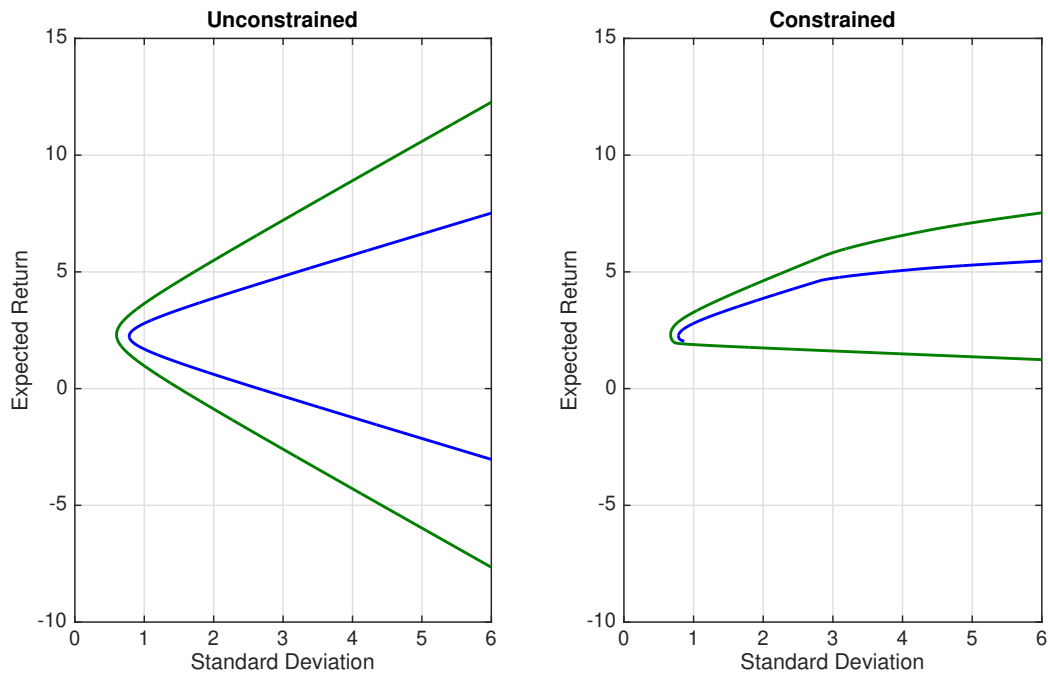
## 4.5 Analysis of the Investable PrEQIn indices

In this section, we consider the performance impact of including a larger number of investable indices. To do so, we consider an investment universe with the base portfolio to which we add the possibility of investing in each of the thirteen PrEQIn indices (see Trojani, Orlowski, Popescu and Sali (2017) for details). The efficient frontiers displayed in Figure 8 are similar to the one reported in the left panel of Figure 4. The short sales constrained efficient frontier extends in the lower part since one of the PrEQIn indices (PrEQIn Early Stage) has a negative average return over the sample period 2001-06-30 to 2016-03-31. We further observe that Figure 9 is similar to the bottom panel of Figure 5 with only a marginal improvement in terms of performance when we compare to the Private Global GMVP portfolio. Therefore diversification benefits do not significantly improve by considering a large number of indices. In fact, the additional performance is obtained by investing in a relatively limited number of indices. Looking at the evolution of portfolio weights in Figure 10, we can see that only three indices have a weight larger than 0 at some point in the sample for the short sales constrained GMVP, these are PrEQInBytSmlCp, PrqnDstrsdPrvtEqty and PrqnExpnsn, which correspond to Buyout Small Cap, Distressed, and Expansion PrEQIn indices. From this, it seems that it might be useful to consider subcategories to fine tune the investment in illiquid asset classes, and even further benefit from diversification opportunities.

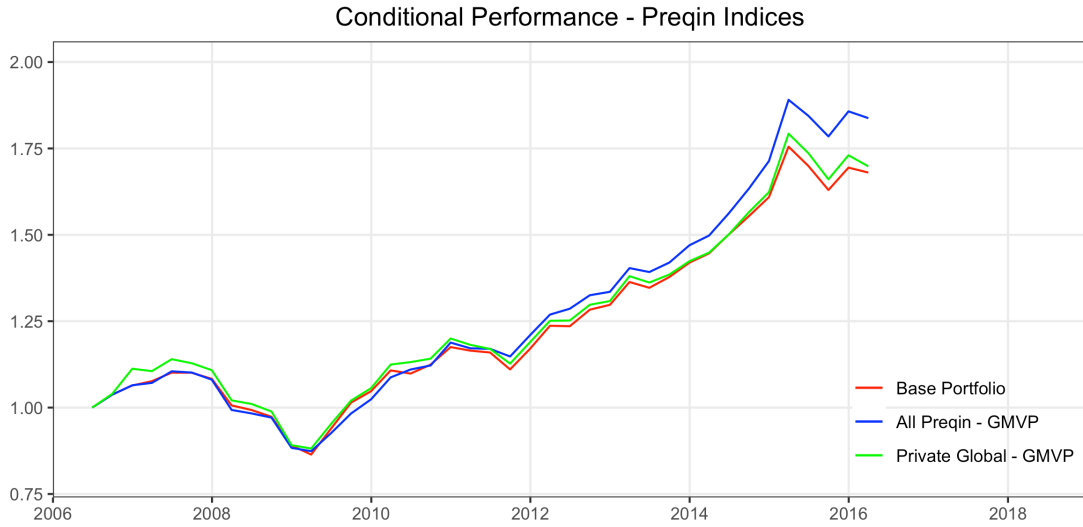




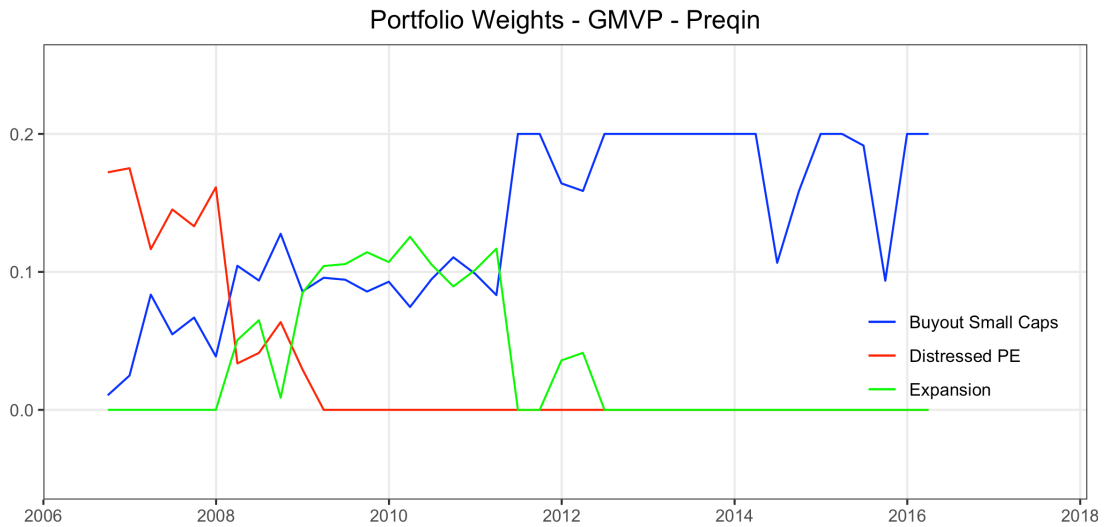
**Figure 7:** Conditional performance for the sample period 2005-03-31 to 2016-09-30. The base portfolio is composed of 50% in MSAINT, 20% in JPMGBE, 20% in WBEM-H and 10% in ECLIT3. It appears in red in both panels. Both panels consider the introduction of the publicly traded illiquid assets for 20% weight and keep the base portfolio 80% fixed allocation. The top panel displays the Tangency strategies across regions and the bottom panel displays the GMVP strategies across regions. The global portfolio includes MSCI ACWI IMI Core Real Estate Index and MSCI ACWI Infrastructure Index. The European portfolio includes MSCI EMU Core Real Estate Index and MSCI EMU Infrastructure Index. The Emerging market portfolio includes MSCI EM Core Real Estate Index and MSCI EM Infrastructure Index.



**Figure 8:** Efficient frontiers for the sample period 2001-06-30 to 2016-03-31. The frontier in blue is the basic stock bond cash allocation using MSAINT, JPMGBE, WBEM-H and ECLIT3. The frontiers in green adds all PrEQIn indices for private equity and real estate. The left panel is unconstrained and the right panel is short sales constrained.



**Figure 9:** Conditional performance for the sample period 2006-09-30 to 2016-03-31. The base portfolio is composed of 50% in MSAINT, 20% in JPMGBE, 20% in WBEM-H and 10% in ECLIT3. The Private Global GMVP adds the privately traded illiquid assets HF global, RE global, PE global. The all PrEQIn GMVP adds all PrEQIn indices for private equity and real estate.



**Figure 10:** Active portfolio weights for the GMVP strategy with short sales constraint for the sample period 2006-09-30 to 2016-03-31. 80% in the base portfolio and 20% allocated through all Preqin indices.

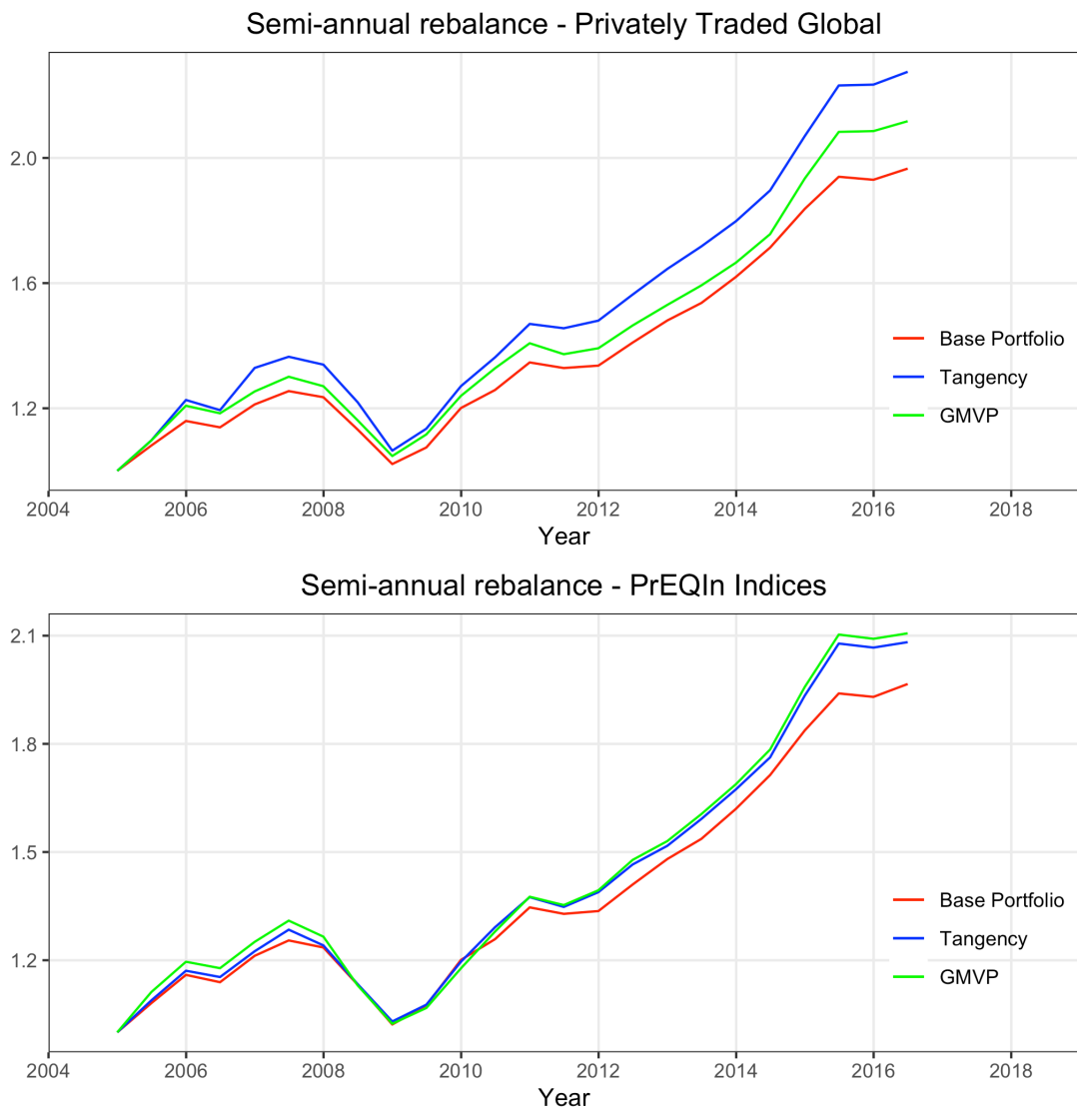
## 4.6 Robustness check with different rebalancing frequencies

In previous sections, we used quarterly rebalancing frequency for all conditional allocations. Given the properties of illiquid assets, the liquidity of specific asset markets may be drained at a quarterly frequency. Therefore, we execute a robustness check by extending the rebalancing cycle to both semi-annual and annual frequencies.

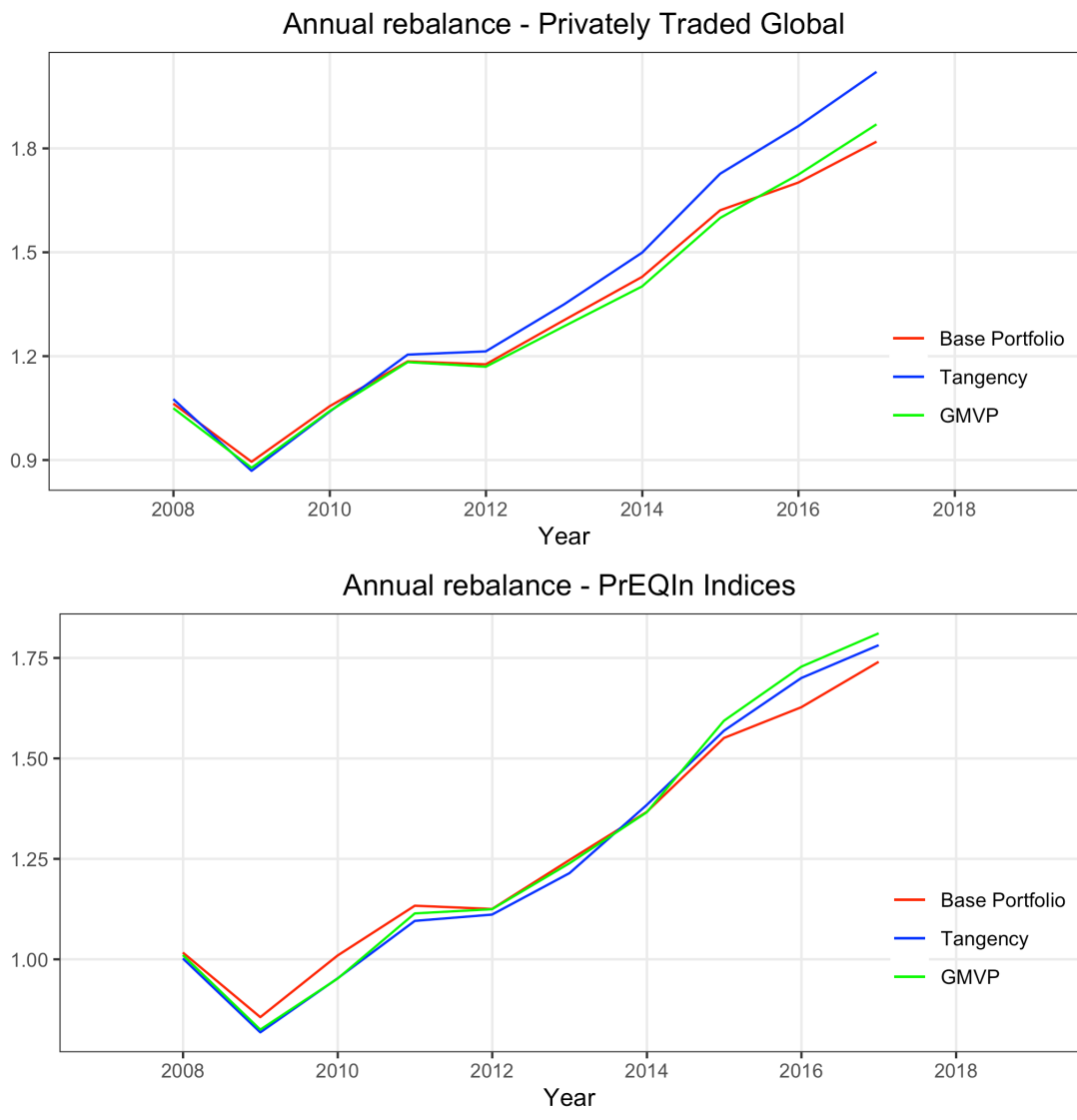
Figure 11 re-examines the dynamic allocations for two sets of assets, privately traded global illiquid assets (HF, RE, and PE) and investable PrEQIn indices, separately with semi-annual rebalancing frequency from 2005 to 2016. Results appear similar to Figure 5, which uses quarterly rebalancing frequency, with the difference that the Tangency portfolio performs better than the GMVP strategy. At the bottom panel of Figure 11, we replace the global MSCI illiquid asset indices with investable PrEQIn indices. The Tangency and GMVP strategies have very close performance, and both of them outperform the base portfolio.

We further extend the impact of the rebalancing cycle by focusing on annual frequency in Figure 12. Overall, the results convey the same message as other rebalancing frequencies. The top panel shows that the Tangency strategy still outperforms the base portfolio significantly, but the GMVP strategy only outperforms the base portfolio by a small margin in long-term investment. Similarly, when we switch to investable PrEQIn indices, both Tangency and GMVP strategies can still slightly beat the base portfolio in the long run.

The benefits from diversification require a readjustment of the portfolio which is barely achieved at annual frequency. The contribution of illiquid assets, in the context of our analysis, becomes less important at a very low rebalancing frequency, but as evidenced by the results in Figure 11, a quarterly rebalancing frequency, which is realistic for the type of assets we consider does yield a significant performance improvement.



**Figure 11:** Conditional performance for the sample period 2005-06-30 to 2016-06-30. The base portfolio is composed of 50 % in MSAINT, 20 % in JPMGBE, 20 % in WBEM-H and 10 % in ECLIT3. Following the core-satellite method, both tangency and GMVP portfolios limit 80% weight in the base portfolio and 20 % weight allocated through illiquid assets with short-sales constraints. In the top panel, the Privately Traded Global portfolios add the privately traded illiquid assets HF global, RE global, PE global. In the bottom panel, we add investable PrEQIn indices. All portfolios are rebalanced at semi-annual frequency.



**Figure 12:** Conditional performance for the sample period 2005-06-30 to 2016-06-30. The base portfolio is composed of 50% in MSAINT, 20% in JPMGBE, 20% in WBEM-H and 10% in ECLIT3. Following the core-satellite method, both Tangency and GMVP portfolios limit 80% weight in the base portfolio and 20% weight allocated through illiquid assets with short-sales constraints. In the top panel, the Privately Traded Global portfolios add the privately traded illiquid assets HF global, RE global, PE global. In the bottom panel, we add investable PrEQIn indices. All portfolios are rebalanced at annual frequency.

## 5 Conclusion

The main message from the above analysis is that illiquid asset classes provide sizable gains in terms of performance when added to standard asset classes. They are clearly non redundant from a diversification perspective. The benefits are especially there for dynamic conservative strategies targeting low volatility. A refined investment strategy based on disaggregate information per category of illiquid asset classes might be worth being implemented since some categories seem to deliver better diversification benefits than others.

## References

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Trojani, F., Orlowski, P., Popescu, A., and A. Sali (2017). "The historical risk-return trade-offs of illiquid asset classes". Natixis White Paper.