B.XI. ARFIMA Models

Olivier Scaillet

University of Geneva and Swiss Finance Institute

Outline

1. Introduction
2. Fractional White Noise
3. ARFIMA(p,d,q)

Introduction

Order of differencing: $(1 - L)^d$

- $d = 1$: I(1) process,
  - ACF declines linearly
- $d = 0$: I(0) process,
  - ACF declines exponentially

What happens if d is non-integer?

$\Rightarrow$ fractionally integrated processes

called *ARFIMA processes*

Fractional White Noise

Fractional white noise: ARFIMA(0,d,0)

$$(1-L)^d Y_t = \varepsilon_t$$

ACF declines hyperbolically:

$$\rho_k = const \times k^{2d-1}$$

- Process is weakly stationary for $d < 0.5$ and invertible for $d > -0.5$.
- Process is nonstationary for $d \geq 0.5$ (infinite variance).

Fractional White Noise

To make concept operational, use binomial series expansion:
For any real $d > -1$

$$(1-L)^d = \sum_{k=0}^{\infty} \begin{pmatrix} d \\ k \end{pmatrix} (-L)^k$$
$$= 1 - dL + \frac{d(d-1)}{2!} L^2 + \frac{d(d-1)(d-2)}{3!} L^3 + \dots$$

Useful model when first differencing (standard arima) takes out too much:
$\Rightarrow$ Over-detrending(-differencing) phenomenon

ARFIMA(p,d,q)

The general ARFIMA(p,d,q) model is given by:

$$\Phi(L)(1-L)^d Y_t = \mu + \Theta(L)\varepsilon_t$$

Estimation of d is complicated and usually done in the frequency domain.