B.VII. Stochastic Volatility Models

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Outline

Stochastic Volatility vs ARCH Models

- In ARCH models,
  - Conditional variance is not constant but marginal variance, i.e. variance of the stationary distribution, is constant.
- In *stochastic volatility models*,
  - The variance itself follows a random process

$$Y_t = \mu + \sigma_t \varepsilon_t.$$

The stochastic volatility model corresponds to the discretisation of a Geometric Brownian motion

$$\frac{dS_t}{S_t} = \mu dt + \sigma_t dW_t$$

with a random diffusion term.

Stochastic Volatility Model Interpretation

This type of process underlies the stochastic volatility extension of the Black-Scholes option pricing model.

Usually a lognormal model is taken for the stochastic variance (Hull and White stochastic option pricing model):

$$h_t = \ln(\sigma_t^2) = a_0 + a_1 h_{t-1} + \eta_t$$

where $\eta_t \sim N(0, \sigma_\eta^2)$. Hence

$$Y_t = \mu + \exp(h_t/2)\varepsilon_t.$$

Empirical Illustration

- Dollar/Sterling exchange rate.
  - From January 1974 to December 1994, 5192 observations.
- Estimation by maximum likelihood:

$$\hat{a}_0 = 1.044, \quad \hat{a}_1 = 0.989, \quad \hat{\sigma}_\eta = 0.125$$