

B.XII. Cointegration

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Introduction

Concept of cointegration depicts long-run relationships between economic variables.

Examples:

- consumption and income,
- long and short term interest rates,
- prices and wages,
- stock prices and dividends.

Aim of cointegration is the detection and analysis of long run relationships among time series variables.

Definition of Cointegration

A vector of n non-stationary $I(1)$ variables $X_t = (X_{1,t}, \dots, X_{n,t})$ is said to be cointegrated if there exists a vector β such that the univariate series $\beta' X_t$ is stationary.

Example:

Let c_t , resp. y_t , be the logarithm of consumption, resp. income. These two series may be considered to be nonstationary $I(1)$ (large fluctuations).

However savings rate $c_t - y_t$ has lower fluctuations and can be expected to be stationary.

Granger's Representation Theorem

(Granger) We may represent the behavior of the consumption as an error correction model:

$$A(L)\Delta c_t = \alpha + B(L)\Delta y_t + \lambda(c_{t-1} - y_{t-1}) + u_t$$

only containing stationary variables.

When $\lambda < 0$ and the disequilibrium $c_{t-1} - y_{t-1}$ is large, the impact on the variation Δc_t will be negative tending to *correct* this disequilibrium.

Cointegration Tests

Tests for presence of cointegration relationships can be based on:

- Stationarity tests on the residuals from a regression among components of X_t (Engle-Granger procedure).
- Estimation by maximum likelihood of a VAR on X_t (Johansen procedure).