B.II. Innovation of a Process

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Outline

Strong Innovation

Let Y be a second order process.

$Y_{\underline{t-1}} = (Y_{t-1}, Y_{t-2}, ...)$ denotes all the past of Y_t.

Definition:
We call *strong innovation* of Y the process defined by

$$\eta_t = Y_t - E\left[Y_t \middle| Y_{\underline{t-1}}\right].$$

It corresponds to the difference between the realization of the process at date t, and its expectation knowing all its past.

Linear Past and Linear Expectation

Definition:

The *linear past* of Y_t is the vector space made of all variables corresponding to linear combinations (finite or infinite) of $1, Y_{t-1}, Y_{t-2}, \dots$

This space is denoted by H_{t-1}.

Definition:

The *linear expectation* of Y_t *conditionally* to its past is the projection of Y_t on H_{t-1}. This expectation is denoted by

$$Y_t^* = EL\left[Y_t \,|H_{t-1}\right]$$

Interpretation:

Y_t^* is the best approximation of Y_t as linear combination of its past.

Linear Innovation and White Noise

Definition:
The *linear innovation* of Y is the process defined by $\varepsilon_t = Y_t - Y_t^*$.

The decomposition $Y_t = Y_t^* + \varepsilon_t$ is called an *affine regression* of Y_t on its past.

This decomposition is unique.

Property: The process (ε_t) is a *white noise*, i.e., it will correspond to a sequence of homoscedastic (same variance), uncorrelated (autocovariances are zero), and centered (mean zero) random variables.