A.V. Application to CAPM

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Outline

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Mean-Variance Frontier

Markowitz (1959) mean-variance frontier:

Investors select their portfolio in terms of the expected return (reward) and the variance (risk) of returns.

Optimally they will hold
a mean-variance efficient portfolio,

i.e. a portfolio with the highest expected return for a given level of variance.

Sharpe-Lintner CAPM

Sharpe (1964), Lintner (1965) CAPM:

If investors have *homogeneous expectations* and optimally hold mean variance efficient portfolios, then in the absence of market frictions:

The portfolio of all invested wealth, or the *market portfolio*, will itself be a mean-variance efficient portfolio, and the following relation holds

$$E\left[Y_{i,t}\right] = r_f + \beta_{im}\left(E\left[Y_{m,t}\right] - r_f\right),$$

with $\beta_{im} = \frac{\text{Cov}(Y_{i,t}, Y_{m,t})}{Var[Y_{m,t}]}$

$Y_{m,t}$ is the return of the market portfolio and r_f is the constant return on the risk free asset (risk free rate).

Testing the CAPM

Excess returns:

Let $Z_{i,t}$ represent the return on the ith asset in excess of the riskfree rate

$$Z_{i,t} = Y_{i,t} - r_f,$$

then more compactly we have:

$$E\left[Z_{i,t}\right] = \beta_{im} E\left[Z_{m,t}\right],$$

with $\beta_{im} = \frac{Cov(Z_{i,t}, Z_{m,t})}{Var[Z_{m,t}]}$.

$Z_{m,t}$ is the excess return on the market portfolio of assets.

Testing the CAPM

The CAPM relation gives a *linear* rule of pricing.

Empirical tests have focused on three implications of CAPM relation:

1. intercept is zero.
2. variation of excess returns between assets is completely captured by betas.
3. market risk premium $E[Z_{m,t}]$ (expected excess return on market) is positive.

Testing the CAPM

All tests of these implications involve OLS estimators of the slope coefficient of the CAPM relationship, i.e. the beta in the regression equation

$$Z_{i,t} = \beta_{im} Z_{m,t} + \varepsilon_{i,t},$$

which is an empirical version of the theoretical CAPM equation.

Most empirical studies based on the use of returns on stock indices as market returns reject CAPM implications.

Roll's (1977) Critique

Roll (1977) critique:

Stock indices are only proxies of the market portfolio.

The market portfolio contains all assets in the economy, including real estate, art, ...

The CAPM might not be rejected if the return on the true market portfolio was used.