Family Name: Geneva, January 13, 2014
Given Name:

Financial Econometrics

The exam consists of **14 questions**, either binary choice or open questions. Binary questions are worth (1) point for a correct answer, (-0.5) points for a wrong answer, and (0) points if left blank.

The exam lasts one hour and thirty minutes **(1h30)**. Note that vague or unclear answers place yourself at a disadvantage.

Please report your name over all pages before you start the exam.

This is a closed-book exam. No study materials and/or calculators are allowed.

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Question 1

Proposition: To test the null hypothesis that an AR(1) process has a unit root, one may use the Student distribution.

- True

- False

Answer: False. When the process has a unit root the standard t-test doesn't apply.

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Question 2

Proposition: An i.i.d. process is always weakly stationary.

- True

- False

Answer: False. An i.i.d. process may have infinite variance.

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Question 3

Proposition: The asymptotic distribution of maxima of every i.i.d. process is described by three limit distributions.

- True

- False

Answer: True. It is the Fisher-Tippett theorem.

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Question 4

Proposition: If the empirical ACF (autocorrelation function) dies off more or less geometrically with increasing lag number, it is a sign that the series obeys a pure AR process.

- True

- False

Answer: True. See TP9.

Question 5

Let $x_1, ..., x_N$ be an i.i.d N-sample of fire losses recorded in Geneva. We assume that fire losses may be modeled as a random variable following an exponential distribution with parameter λ. Detailing the steps involved, derive the likelihood and log-likelihood functions in order to arrive at the maximum likelihood estimator for λ.

Reminder: The density of a random variable following an exponential distribution with parameter λ is given by:

$$f(x) = \left\{ \begin{array}{ll} \lambda e^{-\lambda x} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{array} \right.$$

Answer: likelihood $= \lambda^N e^{-\lambda(\sum_{i=1}^N x_i)}$, log-likelihood $= N log\lambda - \lambda(\sum_{i=1}^N x_i)$, maximum likelihood estimator $\hat{\lambda} = \frac{N}{\sum_{i=1}^N x_i}$.

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Question 6

Let X and Y be two random variables. Let us consider the non-parametric regression

$$Y = m(X) + \epsilon.$$

We observe simultaneously the samples $(Y_1, ..., Y_T)$ and $(X_1, ..., X_T)$. Also, we assume that the support of the random variable X is the set of positive real numbers (as in the case of, for example, insurance losses or bond durations). Write and explain your choice for an estimator $\hat{m}$ of the (possibly non-linear) function m. In particular, specify in detail what kind of kernel would you employ and why.

Answer: Non parametric estimator:

$$E[Y \mid X = x_i] = \hat{m}(x_i) = \frac{\frac{1}{Th} \sum_{t=1}^{T} y_t K\left(\frac{x_t - x_i}{h}\right)}{\frac{1}{Th} \sum_{t=1}^{T} K\left(\frac{x_t - x_i}{h}\right)},$$

K = Kernel (see A.IX). The data are bounded, it is then better to use an asymmetric kernel.

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Question 7

1. Define the concept of Expected Shortfall (ES) of a portfolio and provide its defining formula.

2. What is the main advantage over the Value at Risk (VaR)?

3. Explain how you can calculate the ES using Extreme Value Theory.

Answer: 1) see A.X. 2) subadditivity. 3) see A.XI.

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Question 8

Let $x_1, ..., x_T$ be an i.i.d T-sample of gaussian random variables with distribution $X_t \sim \mathcal{N}(\mu, \sigma^2)$. It follows that the sample mean $\hat{\mu}$ and the sample variance $\hat{\sigma}^2$ satisfy the following properties

$$\sqrt{T}(\hat{\mu} - \mu) \sim \mathcal{N}(0, \sigma^2), \quad \sqrt{T}(\hat{\sigma}^2 - \sigma^2) \sim \mathcal{N}(0, \sigma^4),$$

and $\hat{\mu}$ and $\hat{\sigma}$ are orthogonal.

By using the delta method and the above properties, find the 95% asymptotic confidence interval of $R = \mu^2 \sigma^3$.

Answer: see TP 8.

$\theta = \begin{pmatrix} \mu \\ \sigma^2 \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^4 \end{pmatrix}$. Delta method: $\sqrt{T}(\hat{R} - R) \sim \mathcal{N}(0, \frac{\partial R}{\partial \theta} \sum \frac{\partial R'}{\partial \theta})$.

$\widehat{\frac{\partial R}{\partial \theta} \sum \frac{\partial R'}{\partial \theta}} = \hat{V} = 4\hat{\sigma}^8 \hat{\mu}^2 + \frac{9}{4}\hat{\sigma}^6 \hat{\mu}^4$.

95% asymptotic confidence interval of R: $\left[\hat{R} - \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{T}}, \hat{R} - \frac{z_{\frac{\alpha}{2}}}{\sqrt{T}} \right]$, where $z_{1-\frac{\alpha}{2}}$ and $z_{\frac{\alpha}{2}}$ are quantiles of $\mathcal{N}(0, \hat{V})$.

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Question 9

Following Jensen (1968), suppose you want to test whether a given mutual fund provides consistently higher returns than those implied by the Capital Asset Pricing Model (CAPM). To do so, you estimate α_i and β_i in the linear equation

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + \varepsilon_{i,t},$$

where $R_{i,t}$, $R_{m,t}$, and $R_{f,t}$ are the returns of the mutual fund, the market, and the risk-free rate respectively. $\varepsilon_{i,t}$ is the noise term.

1. State the null hypothesis corresponding to no abnormal returns above those prescribed by the CAPM.

2. How can you test the null hypothesis above?

3. Provide the formula for the Ordinary Least Squares (OLS) estimator of $\hat{\beta}_i$.

Answer: 1) $\alpha_i = 0$. 2) OLS t-test. t-statistic: $\frac{\hat{\alpha}_i}{\hat{\sigma}_{\hat{\alpha}_i}}$. 3) $\widehat{\beta}_i = \frac{Cov[R_{i,t} - R_{f,t}, R_{m,t} - R_{f,t}]}{Var[R_{m,t} - R_{f,t}]}$.

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Question 10

Let $(x_1, .., x_T)$ be observations of the random variable X. Define the function $K(u)$ on the real numbers as:

$$K(u) = \begin{cases} \frac{4}{3}u + \frac{4}{3} & \text{if } -1 \leq u < -\frac{1}{2}, \\ \frac{2}{3} & \text{if } -\frac{1}{2} \leq u \leq \frac{1}{2}, \\ -\frac{4}{3}u + \frac{4}{3} & \text{if } \frac{1}{2} < u \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Provide an argument to show whether or not $\widehat{f}(x) = \frac{1}{Th} \sum_{t=1}^{T} K\left(\frac{x_t - x}{h}\right)$ is a kernel estimator of the density of the distribution of X. If $K(u)$ can be used as a kernel, specify in which context you might find it undesirable to use it.

Answer: K is positive and integrates to one, hence it can be used as kernel. $K(u)$ is symmetric, you shouldn't use it with bounded data.

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Question 11

The following graphs show the sample autocorrelation function and partial autocorrelation function of a simulated series. Horizontal lines correspond to 95% confidence bounds.
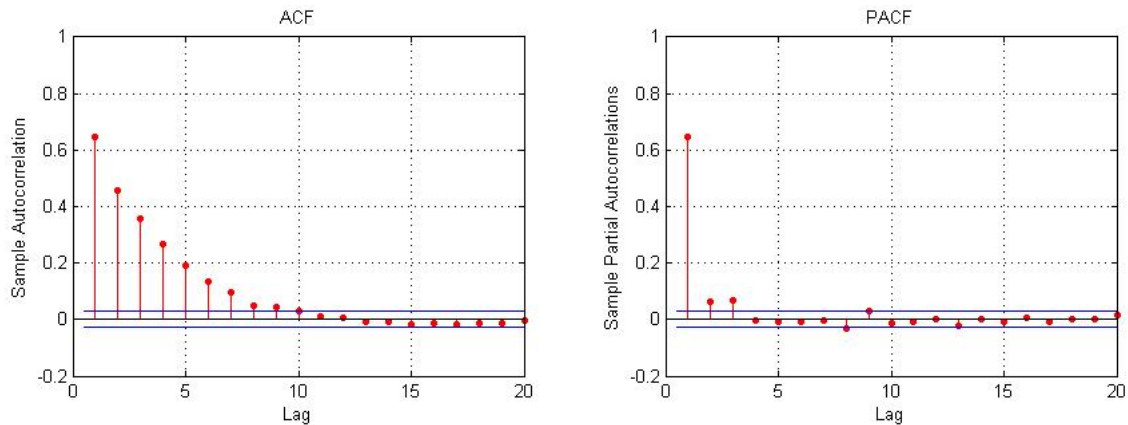

Figure 1: Autocorrelogram (left) and Partial Autocorrelogram (right) of the series

1. Which model might have been used to simulate this series? Provide its theoretical equation, as well as the underlying hypotheses.

2. Which test allows you to determine if the model postulated above is appropriate? Explain how to calculate and interpret the test statistic and provide all the underlying hypotheses.

Answer: 1) AR(3). 2) Ljung box test on residuals.

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Question 12

After determining the order of the model in the previous question, you now use the squared residuals of that model to identify ARCH effects. The following graphs show the correlogram and partial correlogram of the squared residuals, as well as the 95% confidence interval.
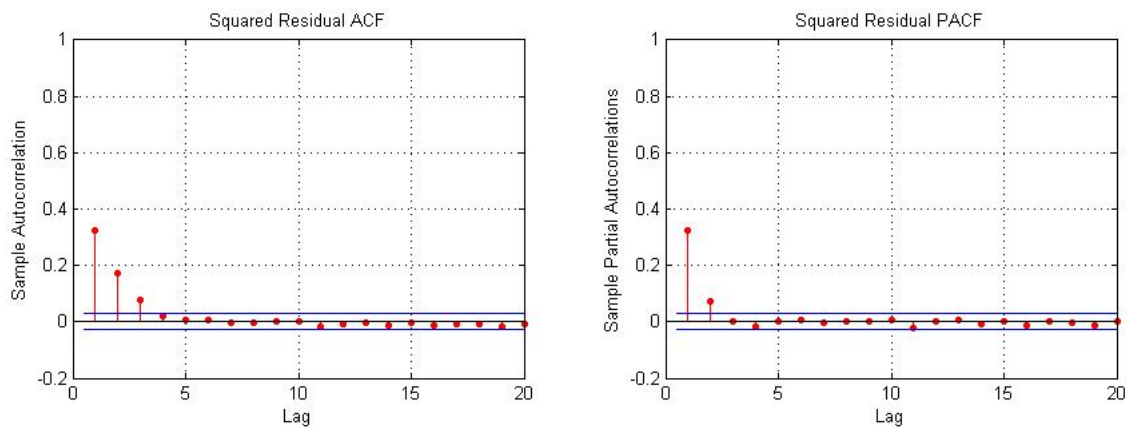

Figure 2: Autocorrelogram (left) and Partial Autocorrelogram (right) of the squared residuals

1. How would you extend the model in the previous question to deal with this additional information? Provide its theoretical equation, as well as the underlying hypotheses.

2. Which test allows you to determine if the model postulated above is appropriate? Explain how to calculate and interpret the test statistic and provide all the underlying hypotheses.

Answer: 1) AR(3)-GARCH(0,2). 2) Arch test on squared residuals.

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Question 13

Suppose that the weekly log-returns r_t of the IBM stock can be modeled with the following AR(1)-GARCH(1,1) model:

$$r_t = \mu + \varphi r_{t-1} + \varepsilon_t,$$

$$\varepsilon_t = \sigma_t z_t,$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \quad \omega > 0, \quad \alpha, \beta \geq 0, \quad \text{and} \quad 1 > \alpha + \beta,$$

where z_t is a gaussian white noise ($z_t \sim \mathcal{N}(0,1)$), $|\varphi| < 1$, and σ_t^2 is the conditional variance of ε_t.

1. Calculate the conditional mean and variance of r_t at time $t-1$ ($E[r_t|I_{t-1}]$ and $V[r_t|I_{t-1}]$).

2. Let us define $\hat{\sigma}_t^2(j) = V[\varepsilon_{t+j}|I_t]$ as the conditional variance of ε_{t+j}. Calculate $\hat{\sigma}_t^2(1)$ and $\hat{\sigma}_t^2(2)$ as functions of $\omega, \alpha, \beta, \sigma_t^2$, and ε_t^2.

Answer: 1) $E[r_t|I_{t-1}] = \mu + \varphi r_{t-1}$, $V[r_t|I_{t-1}] = \sigma_t^2$. 2) $\hat{\sigma}_t^2(1) = \omega + \alpha \varepsilon_t^2 + \beta \sigma_t^2$, $\hat{\sigma}_t^2(2) = \omega + (\alpha + \beta)(\omega + \alpha \varepsilon_t^2 + \beta \sigma_t^2)$.

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Question 14

Explain the Matlab code below. In particular:

1. Indicate in which context this code is used.

2. Indicate which measure the variables x1 and x2 contain at the end of the program.

3. In general, would you expect x1 to be bigger or smaller than x2 and why?

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% FireLosses is a vector containing the  accident
% losses over 300 000 Danish Krone (DKK),
% from the years 1980 to 1990
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mu=mean(FireLosses);
sigma=std(FireLosses);
z_99=norminv(.99,0,1);

x1=mu+sigma*z_99
guess=x1;
x2=fminsearch(@(x) obj_fct(x,FireLosses),guess)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function f=obj_fct(x,z)
    alpha=.01;
    h=std(z)*length(z)^(-.2);
    f=(mean(normcdf((z-x)/h))-alpha)^2;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Answer: see TP 6. 1) Risk analysis, calculation of VaR. 2) $x1 =$ gaussian VaR, $x2 =$ non parametric VaR. 3) In general $x1 < x2$, as the kurtosis of the real distribution is bigger than 3.