Financial Econometrics
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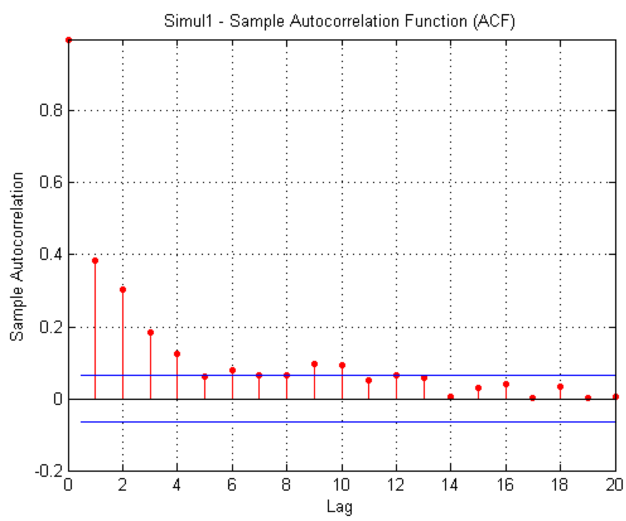
TP 9 – Solution

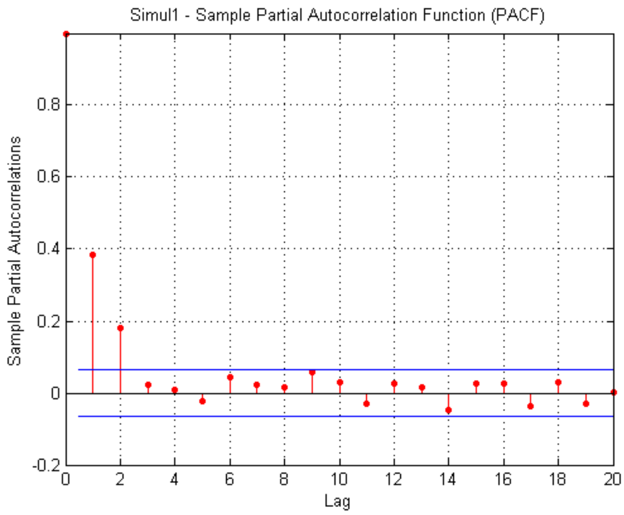
ACF, PACF and ARMA Models

Theory

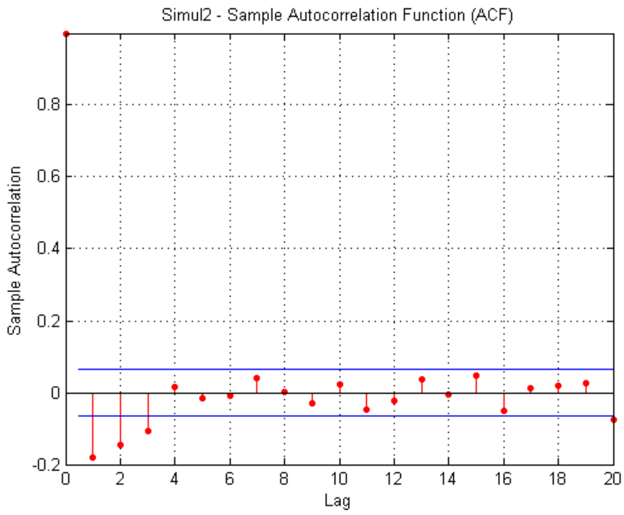
- If the *ACF* dies off more or less geometrically with increasing lag number, it is a sign that the series obeys a *pure* AR process.
- If the *PACF* dies off more or less geometrically with increasing lag number, it is a sign that the series obeys a *pure* MA process.
- If the *ACF* drops to zero after a small number of lags q, it is a sign that the series obeys a MA(q) process.
- If the *PACF* drops to zero after a small number of lags p, it is a sign that the series obeys a AR(p) process.
- If we see no geometric decrease neither in the ACF nor in the PACF, but that there is at least one non zero coefficient, it is a sign that the process *has no pure AR or MA representation.*

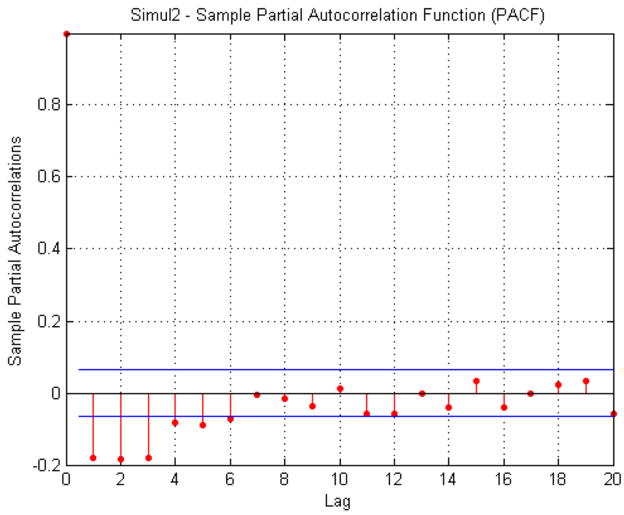
Preestimation Analysis on Simulated Data





The ACF of Simul1 seems to be geometrically decreasing: there are no moving average effects. Its PACF drops to zero after 2 lags: the process may obey an AR(2). (You will see in the next section how to quantify these qualitative checks for correlation using a formal hypothesis test.)

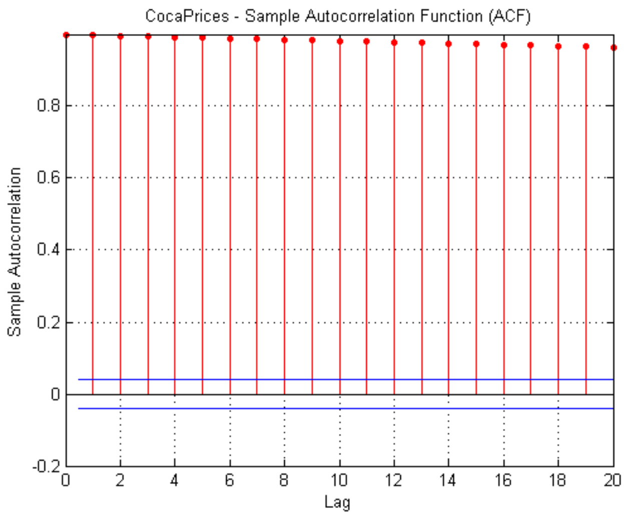


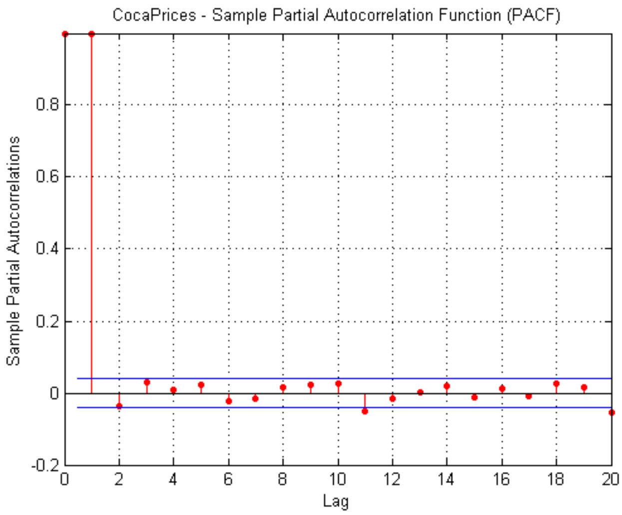


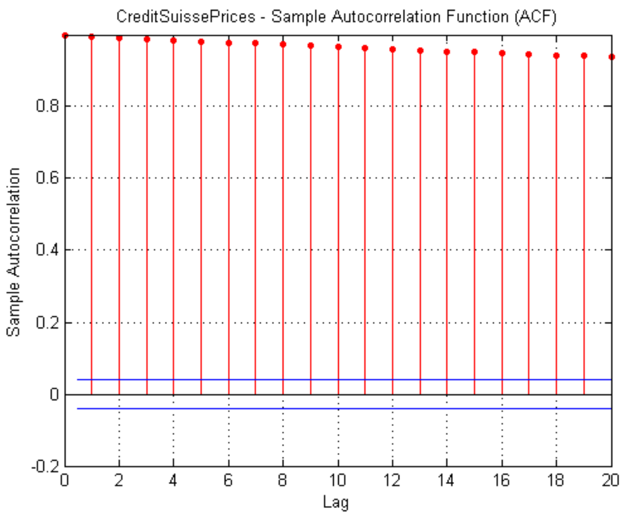
The PACF of Simul2 seems to be geometrically decreasing: there are no autoregressive effects. Its ACF drops to zero after 3 lags: the process may obey an MA(3).

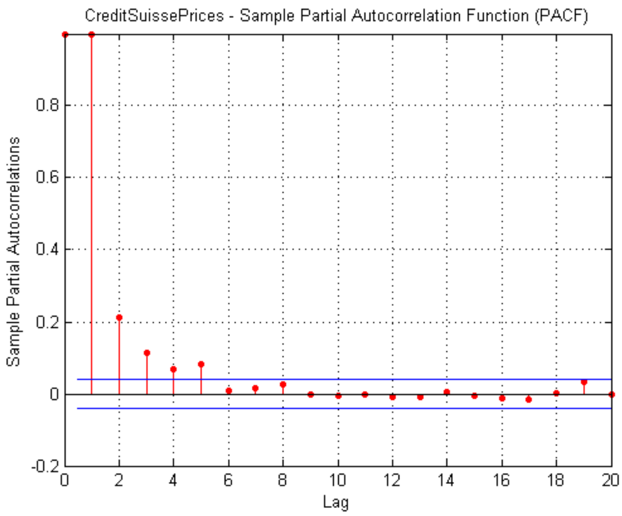
ARMA Representation and Stocks

1. Autocorrelograms and partial autocorrelograms on prices:



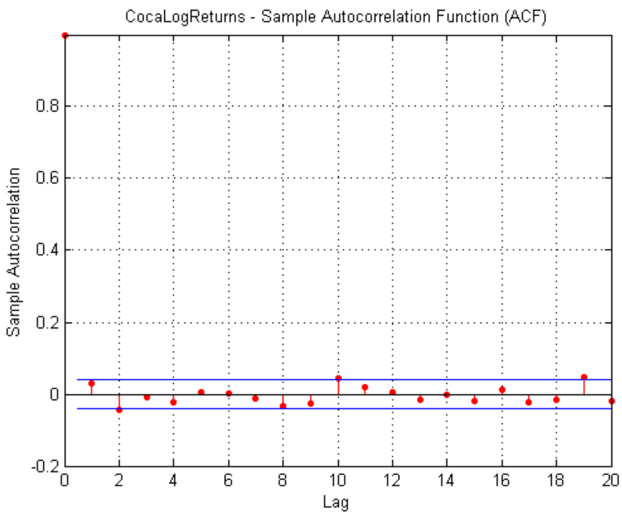


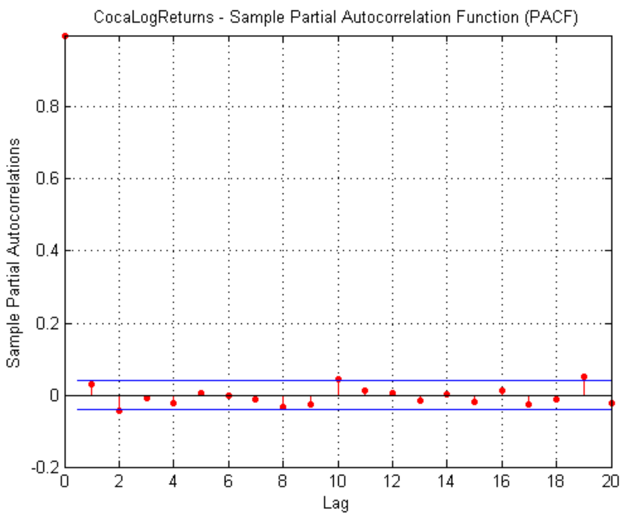


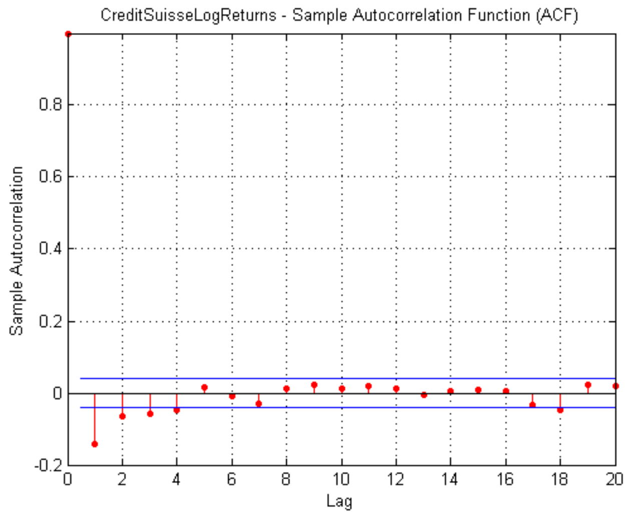


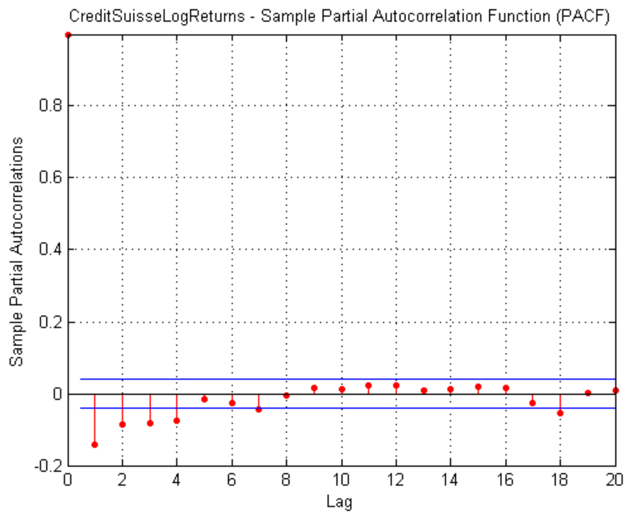
Both series present the same patterns: the lag 1 on the PACF is almost equal to 1 and the ACF is around one for lags greater than or equal to one. Hence, the series are (at least) integrated of order one (unit root problem). Such a process is non stationary. As we do not know how to model non stationary processes, we differentiate the series (or any increasing transformation of the series $\rightarrow$ we differentiate the log of the series which has the property of smoothing the volatility). This is a general fact: all stock price series are integrated of order one.

2. Autocorrelograms and partial autocorrelograms on the first differences of the log-prices (i.e. on logarithmic returns):









3. Results of the Ljung-Box test on Credit Suisse returns:

Lags	H	p-value	Stat	Critical Value
5	1	0	77.35	11.07
10	1	0	81.74	18.31
15	1	0	83.17	25.00
20	1	0	93.53	31.41

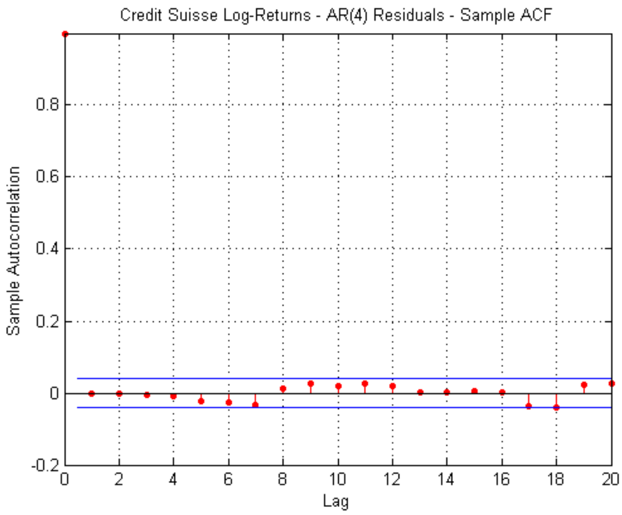
   Hence, there is serial correlation in the returns.

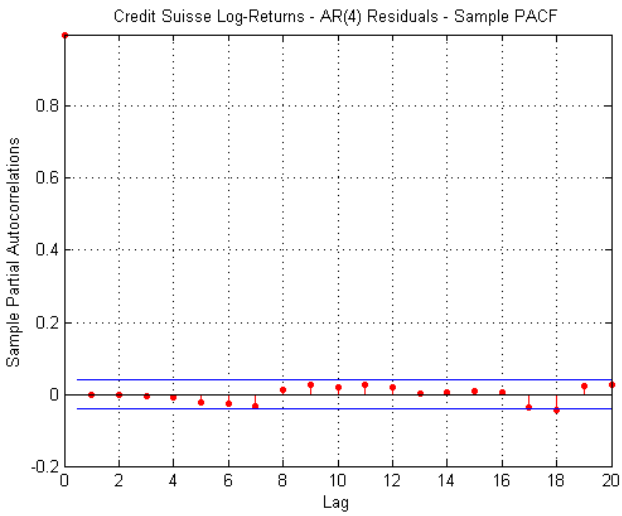
4. As already mentioned, there is no clear pattern which would allow rejecting a moving average representation. Looking at the PACF, we can nevertheless propose an AR(4) representation. (We may need to include the lags 7 and 18.)

   OLS estimates of the coefficients:

	Constant	Lag 1	Lag 2	Lag 3	Lag 4
Value	-0.0004	-0.1658	-0.1067	-0.0940	-0.0757

5. Postestimation Analysis:





Results of the Ljung-Box test on the residuals:

Lags	H	p-value	Stat	Critical Value
5	0	0.91	1.56	11.07
10	0	0.48	9.52	18.31
15	0	0.64	12.57	25.00
20	0	0.24	23.97	31.41

The plots and the Ljung-Box test show that the residuals are not serially correlated. It means that our model is correct, and there is no need for further complication (more particularly the inclusion of lags 7 and 18). We have explained all linearities.

Important remark: When ARMA representations are found for stocks, it is common to see that prices have increased/decreased during a long period. Otherwise, it almost never works!

As a conclusion, pure ARMA models are not good models for financial series. However, adding other features such as ARCH effects to an ARMA representation can seriously improve the overall result!