

TP 3

Asymptotic Properties

Law of Large Numbers

1. Simulate 1000 observations of the laws $\mathcal{U}[0, 6]$ and $\mathcal{N}(3, 5)$.
2. For both series, plot the mean as a function of the sample size n .
($n = 1, 2, \dots, 1000$)
3. Repeat the procedure for the unbiased variance.

Central Limit Theorem

Remember that the central limit theorem establishes that if Y_t is i.i.d. with mean μ and variance σ^2 , then

$$\sqrt{T}(\bar{Y}_T - \mu) \sim \mathcal{N}(0, \sigma^2),$$

where \bar{Y}_T is the sample mean.

1. For the laws $\mathcal{U}[0, 6]$ and $\mathcal{N}(3, 5)$, generate 100 samples of 1000 observations. Then, for each sample, compute the mean on 20, 50, 100, 500 and 1000 observations.
2. For each law and each sample size, estimate the density of the sample mean. Compare the densities on a graph, adding the asymptotic normal distribution.

How many observations do we need in order to obtain good estimates of the parameters and be able to draw correct confidence intervals?