Financial Econometrics
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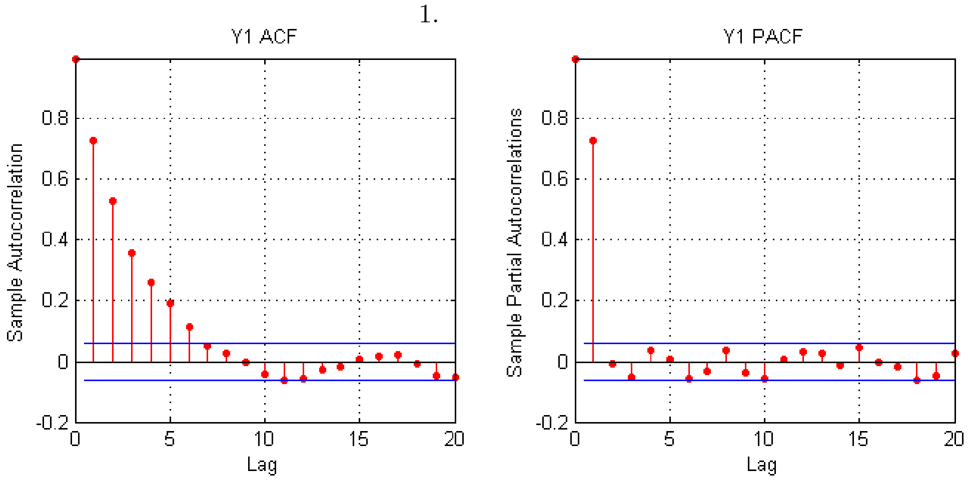
TP 10 – Solution

Models with ARCH Effects

Details are shown only for the series $Y1$.

Preestimation Analysis

1. 

The correlograms indicate that $Y1$ follows an AR(1).

2. Estimates of the AR(1) coefficients:

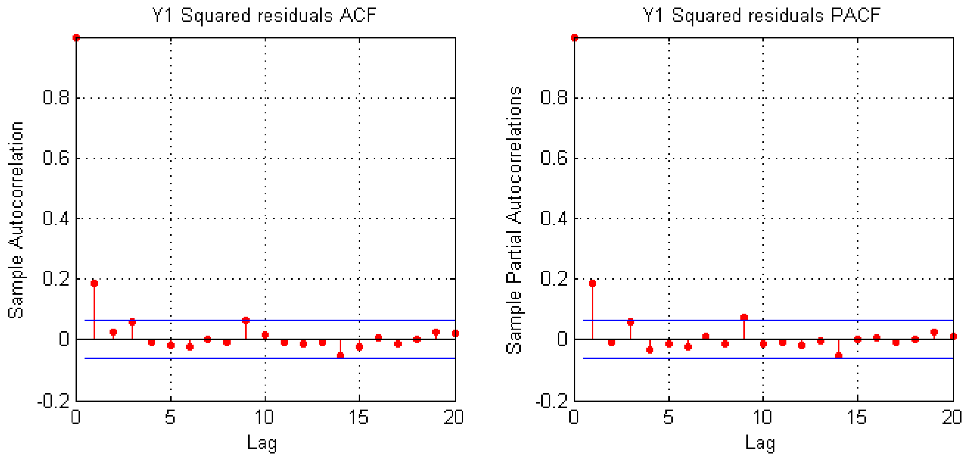
	μ	ω_1
Value	0.82046	0.73067
SE	0.068024	0.018169

Results of the Ljung-Box test on the AR(1) residuals:

Lags	H	pValue	Stat	Critical Value
5	0	0.3459	5.6110	11.0705
10	0	0.3876	10.6232	18.3070
15	0	0.3689	16.2000	24.9958
20	0	0.2651	23.4957	31.4104

3. Results of the ARCH test on AR(1) residuals:

Lags	H	pValue	Stat	Critical Value
5	1	0.0000	39.5998	11.0705
10	1	0.0000	44.8900	18.3070
15	1	0.0000	47.2421	24.9958



The ACF/PACF and the ARCH test show significant evidence in support of GARCH effects (i.e. heteroscedasticity).

4. The following lines show how to combine the equations of pure ARMA and pure GARCH models to obtain the formulation of an ARMA(p_1,q_1)-GARCH(p_2,q_2):

$$\begin{aligned} y_t &= \mu + \sum_{i=1}^{p_1} \omega_i y_{t-i} + \varepsilon_t - \sum_{j=1}^{q_1} \theta_j \varepsilon_{t-j}, \\ \varepsilon_t &= \sqrt{h_t} v_t, \\ h_t &= c + \sum_{j=1}^{q_2} a_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p_2} b_j h_{t-j}, \\ v_t &\sim \mathcal{N}(0,1). \end{aligned}$$

The GARCH(p_2,q_2) part can be rewritten as an ARMA model on squared y's errors (not directly on squared observations):

$$\varepsilon_t^2 = c + \sum_{j=1}^{\max(p_2,q_2)} (a_j + b_j)\varepsilon_{t-j}^2 + u_t - \sum_{j=1}^{q_2} b_j u_{t-j}, \text{ where } u_t = \varepsilon_t^2 - h_t$$

Hence, once we have estimated the ARMA coefficients, we can try to identify q_2 with the ACF of the squared residuals (ε_t^2), and $\max(p_2, q_2)$ with the PACF.

The ACF shows that lag 1 (and maybe 3 and 9) is significant. Given that the PACF does not provide more information than $p_2 \leq q_2$, we will first try an GARCH(0,1). If signs of heteroscedasticity remain, we will use an GARCH(1,1).

Parameter Estimation

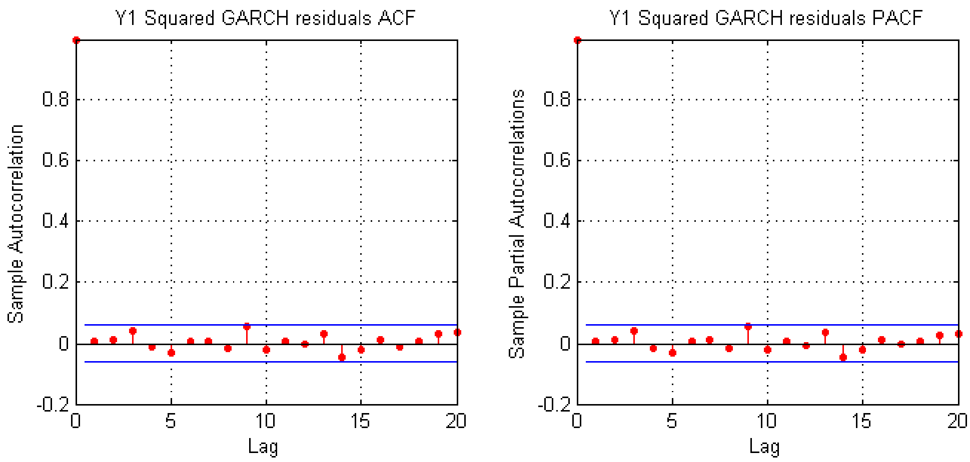
1. Estimates of the GARCH(0,1) coefficients:

	c	a_1
Value	0.99175	0.31103
SE	0.066777	0.047843

Postestimation Analysis

1. Results of the ARCH test on AR(1)-GARCH(0,1) residuals:

Lags	H	pValue	Stat	Critical Value
5	0	0.6404	3.3879	11.0705
10	0	0.6758	7.5175	18.3070
15	0	0.7313	11.2965	24.9958



The tests show that the AR(1)-GARCH(0,1) model sufficiently explains the heteroscedasticity. (We don't need to use a GARCH(1,1).)

Series $Y1$ has been simulated with an AR(1)-GARCH(2,3).

Series $Y2$

For $Y2$ you should find an AR(2) with no GARCH effects. The series has been obtained by simulating an ARMA(2,2). Hence, you see that the method is not perfectly accurate but allows to find the general shape of the model.