B.IV. Nonlinear AR Models

Olivier Scaillet

University of Geneva and Swiss Finance Institute

Outline

AR(p) Model

The linear Autoregressive Process AR(p) is given by

$$Y_t = \mu + \omega_1 Y_{t-1} + ... + \omega_p Y_{t-p} + \varepsilon_t$$

The conditional mean is linear

$$E\left[Y_t \,|\, Y_{t-1}, ..., \; Y_{t-p}\right] = \mu + \omega_1 Y_{t-1} + ... + \omega_p Y_{t-p}.$$

The conditional variance is constant equal to the variance σ^2 of the noise (homoscedasticity)

$$V\left[Y_t \,|\, Y_{t-1}, ..., \; Y_{t-p}\right] = \sigma^2$$

Extensions

Extensions in two directions:

1. Conditional variance may depend on past values.
2. Nonlinearities taken into account

Nonlinear autoregressive process:

$$Y_t = m(Y_{t-1}, ..., Y_{t-p}) + \sigma(Y_{t-1}, ..., Y_{t-p})\varepsilon_t$$

Nonlinear autoregressive (AR) formulation allows for leptokurticity (fat tails).

The conditional mean and variance are unspecified functions of past values.

Non-Linear AR Process: Estimation

We may use nonparametric methods to estimate the conditional mean and variance.

Indeed, at point $Y_{t-1} = y_1, ..., Y_{t-p} = y_p$, the functions

$$\begin{aligned} m(y_1, ..., y_p) =& E\left[Y_t \,|\, Y_{t-1} = y_1, ..., Y_{t-p} = y_p\right] \\ \sigma^2(y_1, ..., y_p) =& V\left[Y_t \,|\, Y_{t-1} = y_1, ..., Y_{t-p} = y_p\right] \end{aligned}$$

may be estimated by a kernel method.

Kernel Estimation

Kernel estimators are given by:

$$\hat{m}(y_1, ..., y_p) = \frac{\frac{1}{Th^p} \sum_{t=1}^{T} y_t \prod_{j=1}^{p} K\left(\frac{y_{t-j} - y_j}{h}\right)}{\frac{1}{Th^p} \sum_{t=1}^{T} \prod_{j=1}^{p} K\left(\frac{y_{t-j} - y_j}{h}\right)},$$

$$\hat{\sigma}^2(y_1, ..., y_p) = \frac{\frac{1}{Th^p} \sum_{t=1}^{T} (y_t)^2 \prod_{j=1}^{p} K\left(\frac{y_{t-j} - y_j}{h}\right)}{\frac{1}{Th^p} \sum_{t=1}^{T} \prod_{j=1}^{p} K\left(\frac{y_{t-j} - y_j}{h}\right)} - (\hat{m}(y_1, ..., y_p))^2$$

Again, extremely simple to implement since they only involve empirical averages.

Application: Spread Dynamics

Application:

Estimation of spread dynamics
Sample: Jan. 1986 to March 2000 (3561 obs.)

- Moody's indices for corporate bond yields with AAA and BAA ratings
- 10 year treasury yield constructed by Federal Reserve bank
- spreads = differences

$$S_t^{AAA} = Y_t^{AAA} - Y_t$$
$$S_t^{BAA} = Y_t^{BAA} - Y_t$$

Application: Spread Dynamics

Statistics:

Spread	$S_t^{AAA} = Y_t^{AAA} - Y_t$	$S_t^{BAA} = Y_t^{BAA} - Y_t$
mean	1.04%	1.91%
st. dev	0.28%	0.38%
min.	0.31%	1.16%
max.	1.96%	3.16%
skew.	0.363	0.751
kurt.	2.719	3.007
corr.	75%	