

## B.XII. Cointegration

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# Introduction

Concept of cointegration depicts long-run relationships between economic variables.

## Examples:

- consumption and income,
- long and short term interest rates,
- prices and wages,
- stock prices and dividends.

Aim of cointegration is the detection and analysis of long run relationships among time series variables.

# Definition of Cointegration

A vector of  $n$  non-stationary  $I(1)$  variables  $X_t = (X_{1,t}, \dots, X_{n,t})$  is said to be cointegrated if there exists a vector  $\beta$  such that the univariate series  $\beta' X_t$  is stationary.

## Example:

Let  $c_t$ , resp.  $y_t$ , be the logarithm of consumption, resp. income. These two series may be considered to be nonstationary  $I(1)$  (large fluctuations).

However savings rate  $c_t - y_t$  has lower fluctuations and can be expected to be stationary.

# Granger's Representation Theorem

(Granger) We may represent the behavior of the consumption as an error correction model:

$$A(L)\Delta c_t = \alpha + B(L)\Delta y_t + \lambda(c_{t-1} - y_{t-1}) + u_t$$

only containing stationary variables.

When  $\lambda < 0$  and the disequilibrium  $c_{t-1} - y_{t-1}$  is large, the impact on the variation  $\Delta c_t$  will be negative tending to *correct* this disequilibrium.

# Cointegration Tests

Tests for presence of cointegration relationships can be based on:

- Stationarity tests on the residuals from a regression among components of  $X_t$  (Engle-Granger procedure)
- Estimation by maximum likelihood of a VAR on  $X_t$  (Johansen procedure)