A.VI. Application to APT

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Outline

Ross APT

Ross (1976) APT:
This is an a alternative pricing theory w.r.t. CAPM, which includes the possibility of multiple risk factors, not necessarily identified with the market portfolio

Assumptions:

1. competitive and frictionless markets
2. generating process for asset returns is of a regression type

$$Y_{i,t} = a_i + b_i' f_t + \varepsilon_{i,t}, E\left[\varepsilon_{i,t} \left| f_t \right.\right] = 0, E\left[\varepsilon_{i,t}^2\right] = \sigma_i^2.$$

The $K \times 1$ vector b_i corresponds to the vector of sensitivities (*factor loadings*) for asset i to the K factors f_t.

Factor Approximation

If the disturbance terms $\varepsilon_{i,t}$ are sufficiently uncorrelated between assets, the absence of arbitrage in large economies (when the number of assets is large) implies that

$$E\left[Y_t\right] \approx \iota\, r_f + B\lambda_K,$$

where

ι is a vector of ones,
r_f is the risk free rate,
$B = \begin{pmatrix} b_1 & b_2 & \cdots & b_n \end{pmatrix}'$,
λ_K is a $K \times 1$ vector of factor risk premia.

The theoretical relation is an approximate one.

Factor Pricing

In practice one assumes an exact factor pricing

$$E\left[Y_t\right] = \iota\, r_f + B\lambda_K.$$

Usually one of the factors is taken as the market portfolio.

Other factors may be portfolios of assets, exchanges rates, oil prices, or macroeconomic variables such as changes in GDP, unemployment rate, inflation, . . .

Once the factors are selected the factor sensitivities can be estimated by OLS from regressing the asset returns on factor data.

Unspecified Factors

Remark:

If the factors are left unspecified (not observable), they may be estimated from statistical approaches known as

factor analysis

and *principal component analysis*.

Both techniques rely on use of the covariance matrix of asset returns while factors can be identified to particular linear combinations of asset returns.

These factor portfolios are called *mimicking portfolios*, because jointly they are maximally correlated with the factors.