Stochastic Processes in Finance
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HW 4

Poisson and Point Processes

Exercise 1

The number of call (put) options exercised per unit of time on a financial exchange is λ (μ). Both counting processes are Poisson ones and independent of each other.

1. Show that the process counting the total number of exercised options is a Poisson one with γ intensity, where $\gamma = \lambda + \mu$ (*Superposition property*).

2. Assume the total number of exercised options follows a Poisson process with γ intensity. If an option is exercised, it is a call with the probability p and a put with $1 - p$ (the probabilities do not depend on previous exercises). Show that the number of exercised calls is described by a Poisson process with $(p\gamma)$ intensity.

3. Find $\mathbb{E}[N_t^2]$, where N_t is a Poisson process with λ parameter.

4. Show that for $t > s$, we have:

$$\mathbb{E}[(N_t - N_s)^2] = \lambda(t - s) + \lambda^2(t - s)^2.$$

Exercise 2

Clients arrive at the shop according to the Poisson process with the intensity of 30 customers per hour. Find the probability that the interval between two successive arrivals is:

1. larger than 2 minutes

2. lower than 4 minutes

3. between 1 and 3 minutes.

Exercise 3

Consider ill patients that go to the hospital. The arrivals follow the Poisson process with the intensity of 1/10 per minute. The hospital does not receive patients until there are at least three people in the waiting room.

1. Determine the average waiting time until the first patient is taken care by the hospital.

2. What is the probability that nobody is received for 60 minutes?

Exercise 4

On a one-way road, the flow of cars is described by the Poisson process with the parameter of 1/6 per second. A pedestrian willing to cross the road needs at least 4 seconds between two successive cars.

1. Compute the probability that he has to wait.
2. Compute the average duration of a time interval between two successive cars that allows him to cross the road.

Exercise 5

Let $\{N_t, t \geq 0\}$ be a Poisson process such that the average number of events on $[0, t]$ time interval is equal to λt.

1. Verify the *absence of memory* property for the time interval between two successive events.
2. Assume the value at t of the counter is equal to 1. Let τ denotes the time of the next arrival.

   Compute $P[\tau < s \mid N_t = 1]$, where $s > t$.

Exercise 6

A factory produces cookies containing chocolate fragments. The number of fragments is a Poisson *point process* with the concentration rate $\lambda = 0.08$ per cm^2. The cookies are rectangular and have the size of 3×4 cm^2. Clients call the claims department if they have no fragment in a cookie.

1. What is the probability to find no fragment in a cookie?
2. What should be the value of λ in order for the complaint rate to be smaller than 1%?

Exercise 7

During a bombing, the fall of bombs is a Poisson point process with the bombing rate $\lambda = 10$ bombs per km^2.

1. What is the probability to find no bomb in a circle of $350m$ radius?
2. What is the average distance to a bomb (being at a random place)?

   Hint: For a continuous rv R which takes positive values $R > 0$, we have $\mathbb{E}[R] = \int_0^\infty P(R > r)dr$.
3. What should be the bombing rate to get the average distance to a bomb of $100m$?